Unitary and Non-unitary Differential Space-Frequency Coded OFDM

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***Abstract*— In this paper, we present the code design structure of unitary and non-unitary *differential space-frequency group codes* (DSFCs) for multiple-input multiple-output (MIMO)-orthogonal frequency division multiplexing (OFDM) systems based on optimal coherent space-frequency (SF) group codes. Under the assumption that the transmitter knows only the delay profile of the channel, a differential transmission rule incorporated with subcarrier allocation is obtained that allows data to be sent without channel estimates at the transmitter or receiver. The differential encoding/decoding is performed in the frequency domain within each single OFDM symbol. Therefore, proposed DSFCs can be successfully decoded even for a rapidly fading channel which may change independently from one OFDM symbol to another. Unitary and nonunitary DSFCs, that are constructed based on design criteria, are compared with recently published techniques in the literature and shown to inherit coding gain of optimal coherent SF codes. Due to design structure and energy constraints, our nonunitary DSFCs do not blow up or diminish during the differential encoding.**

***Index Terms*— Differential modulation, multiple-input multiple-output (MIMO)-orthogonal frequency division multiplexing (OFDM) systems, diversity order, coding gain, unitary and nonunitary group code design, frequency-selective fading channels.**

I. Introduction

Multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) transmission are core technologies for fourth generation wireless systems to improve the capacity and reliability for fading channels. Recently, there has been considerable interest in exploiting spatial diversity through transmit diversity and space-time-frequency coding for MIMO systems that employ multiple transmit and/or receive antennas. In particular, space-frequency coding has emerged as a promising technique for realizing spatial and frequency diversity gains in fast fading broadband wireless systems [1]. Most of the work on space-frequency coding has assumed availability of perfect channel fading estimates at the receiver. This requires power and bandwidth overhead as well as additional hardware to send training symbols or pilot tones to enable the estimation of channel fading parameters on each subcarrier in MIMO-OFDM systems. Furthermore, rapid fading conditions or increasing the number of antennas, make it increasingly more difficult and costly to perform accurate channel estimation at the receiver. Therefore, efficient coding and modulation schemes that do not require channel estimates at the transmitter or receiver are of great interest. For space-time and space-frequency diversity in multiple transmit antennas, differential and noncoherent schemes were proposed in [2]-[14].

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Noncoherent MIMO-OFDM systems are analyzed in [2]. Noncoherent space frequency codes are not particular to differential transmission scheme and are exponential in the code rate. Therefore, implementing those codes requires complex multiplications for high code rates. In [3], spatial diversity achieving differential space-time coding systems was proposed but this scheme does not exploit the additional degrees of freedom offered by multipath propagation i.e. discards the frequency diversity which is available in the wideband frequency-selective fading channels. Schemes that achieve spatial, temporal and frequency diversity in MIMO-OFDM for differential encoding and decoding over spatial, frequency and temporal domains are investigated in [4] [5]. Differential space-time modulation with transmit diversity based on group codes were proposed [6] [7]. The group property of these codes makes it easier to analyze the structures, and yields simpler modulation and demodulation schemes. However, differential space–time modulation is sensitive to interference and is likely to deteriorate or even break down in an interference environment [8]. Differential schemes in frequency domain can be performed by differential encoding/decoding over adjacent subcarriers within each OFDM symbol [9]. However, code construction was not done according to pairwise error probability analysis [9].

In this paper, the design approach followed in [1] for coherent reception is extended to differential case when the channel estimates are not available at the transmitter and receiver (see also [10]). *Differential space-frequency group codes* (DSFCs) for MIMO-OFDM systems are analyzed in a frequency-selective fast fading channel, where the channel is constant within each single OFDM block, and may change independently from one OFDM block to another. The differential schemes for MIMO-OFDM systems proposed in literature rely on the assumption that the fading channel keeps constant within a period of several OFDM blocks, or it changes slowly from a period of several OFDM blocks to another. These schemes cannot successfully decode in rapidly fading channel. In our proposed DSFCs, the transmitted signals are differentially encoded in the frequency domain within each OFDM block such that differential decoding can be performed over subcarriers within each single OFDM block similar to [9]. Differential coding over orthogonal subcarriers can also mitigate the effect of interference compared to differential space-time modulation schemes. Furthermore, the proposed differential SF group codes can be tailored such that only a subset of the available subcarriers are modulated in an opportunistic way to increase the performance of the system significantly compared to block codes if we have some statistical properties like power delay profile (PDP) of the channel at the transmitter. This process is called the *pruning process* where a smooth logical channel is created over subcarriers such that only the “good” part of the channel is used and the “bad” part of the channel is eliminated [9]. DSFCs that are investigated in this paper have a simple maximum likelihood (ML) receiver. The interleaver design or subcarrier allocation method is also embedded in the DSFC design criteria similar to [1]. The choice of subcarrier allocation vector is not chosen in an *ad hoc* fashion but justified by algebraic manipulations. This subcarrier allocation method is based on the assumption of only apriori knowledge of delay profile of the channel at the transmitter, but not exact CSI.

The remainder of this paper is organized as follows. In Section II, the MIMO-OFDM system, criteria for diversity order or coding gain for code design are presented. In Section III, according to design criteria, DSFCs are developed. In Section IV, simulation results and comparison for the proposed code are presented and conclusions are drawn in Section V.

Notation: $\mathcal{E}$ denotes the expectation. Tr($\mathbf{X}$) denotes the trace of the matrix $\mathbf{X}$. $|\mathbf{X}|$ is the determinant of the matrix $\mathbf{X}$. $\|\mathbf{X}\|_F^2$ denotes the Frobenius norm of $\mathbf{X}$ which is $\|\mathbf{X}\|_F^2 = Tr(\mathbf{X}\mathbf{X}^\dagger) = Tr(\mathbf{X}^\dagger\mathbf{X})$. $\mathbf{I}_x$ is the identity matrix of order x. $\mathbf{1}_{x,y}$ is the all-one matrix of size $x \times y$, and $\mathbf{0}$ is the all-zero matrix. $(.)^T, (.)^*$, and $(.)^\dagger$ denote transpose, complex conjugate, and complex conjugate transpose respectively.

II. MIMO OFDM System

A. Frequency-Selective Rayleigh Fading Channel Model

In this paper, we consider a MIMO wireless communication system with T transmit antennas and R receive antennas. We assume that the antennas are spaced at sufficient distance such that the channel between transmitter-receiver pair is quasi-static, wide-sense stationary uncorrelated scattering channel, whose impulse response is

$$h_{t,r}(\tau) = \sum_{l=1}^{L} h_{l,t,r}\,\delta(\tau - \tau_{l,t,r}), \qquad (1)$$

where $L = \lfloor B\tau \rfloor + 1$ is the total number of taps with B and τ denoting the signal bandwidth and delay spread, $h_{l,t,r}$ and $\tau_{l,t,r}$ denote the lth path complex gain and time delay from transmitter t to receiver r, respectively. We also assume that the time delay $\tau_{l,t,r}$ and the variance $\alpha_{l,t,r}^2$ of the lth path are the same for every transmitter-receiver link pair (t,r), hence we can rewrite them as τ_l and α_l^2, respectively, by omitting the transmitter-receiver index (t,r). The L multipath powers are normalized such that $\sum_{l=1}^{L}\alpha_l^2 = 1$.

The channel frequency response is the Fourier transform of the impulse response in (1)

$$H_{t,r}(f) = \sum_{l=1}^{L} h_{l,t,r} e^{-j2\pi f\tau_l}. \qquad (2)$$

Assuming that the channel gain from transmit antenna t to receive antenna r for the nth subcarrier remains constant during the transmission of one OFDM symbol, it can be written as

$$\begin{aligned} H_{t,r}[n] = H_{t,r}(n\Delta f) &= \sum_{l=1}^{L} h_{l,t,r}\, e^{-j2\pi n\tau_l/T_u} \\ &= \sum_{l=1}^{L} h_{l,t,r}\, e^{-j\frac{2\pi}{N}n\beta_l}. \end{aligned} \qquad (3)$$

At the output of the OFDM demodulator, discrete-time frequency response of the channel in (3) can be expressed in matrix form as

$$\mathbf{H}(n) = \sum_{l=1}^{L} \mathbf{H}_l e^{-j\frac{2\pi}{N}n\beta_l}, \quad n = 0, \ldots, N-1, \qquad (4)$$

where $\mathbf{H}(n)$ and $\mathbf{H}_l$ are both RxT, $\mathbf{H}_l$ is the channel impulse response for the lth path and we assume that the elements of $\mathbf{H}_l$ for $l = 0, 1\cdots, L-1$ are circularly symmetric zero-mean uncorrelated complex Gaussian random variables with variance σ_l^2, $\beta_l = \tau_l/T_p$, T_u is the inverse carrier spacing or FFT interval, $\Delta f = 1/T_u$ is the subcarrier spacing, $T_p = T_u/N$ is the sampling period, and N denotes the number of subcarriers of the OFDM block. Note that the duration of one OFDM block is $T_s = T_u + T_G$, where T_G is the duration of guard interval.

B. Design Criteria

In the following, we consider the situation when neither the transmitter nor the receiver has any knowledge of the CSI.

Theorem : Let unitary $Q \times T$ matrix $\mathbf{C_G} \triangleq G\mathbf{D}$ (note that in a unitary group code $\mathbf{C_G}$, $Q = T$ [7]), the pairwise error probability $P(\mathbf{C}_G \rightarrow \mathbf{C}_{G'})$ for the differential transmission when the receiver erroneously decodes $\mathbf{C}_{G'}$ when $\mathbf{C}_G$ is actually transmitted is bounded by

$$P(\mathbf{C}_G \rightarrow \mathbf{C}_{G'}) \le \prod_{r=1}^{R}\prod_{i=1}^{rank(\mathcal{V})} \frac{1}{1 + \frac{T\rho^2}{2(1+2T\rho)}\lambda_i(\mathcal{V})}, \qquad (5)$$

where

$$\mathcal{V} = \frac{1}{2}\mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G - \mathbf{C}_{G'}) \cdot \mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G - \mathbf{C}_{G'})^\dagger. \qquad (6)$$

Rank($\mathcal{V}$) is the rank of $\mathcal{V}$, $\lambda_i(\mathcal{V})$ is the i'th eigenvalue of the matrix $\mathcal{V}$ and ρ is the SNR value. The $Q \times TL$ matrix

$\mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G)$ has the following form [1, Eq. (13)]:

$$\mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G) = \begin{bmatrix} \alpha_1\mathbf{A}_s^{\beta_1}\mathbf{C}_G & \alpha_2\mathbf{A}_s^{\beta_2}\mathbf{C}_G & \cdots & \alpha_L\mathbf{A}_s^{\beta_L}\mathbf{C}_G \end{bmatrix}, \tag{7}$$

where $\mathbf{A}_s$ is the $Q \times Q$ *delay rotation matrix* which is a function of the subcarrier allocation vector s,

$$\mathbf{A}_s = \begin{bmatrix} e^{-j\frac{2\pi}{N}s_0} & & & \\ & e^{-j\frac{2\pi}{N}s_1} & & \\ & & \ddots & \\ & & & e^{-j\frac{2\pi}{N}s_{Q-1}} \end{bmatrix}. \tag{8}$$

Proof: See Appendix. ■

The codeword matrix in the Chernoff bound in ST codes [6] [7] is replaced by the matrix $\mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G)$ which is the *Krylov codeword* associated with the codeword $\mathbf{C_G}$. The Krylov codeword and thus the upper bound on the pairwise error probability (PEP), is a function of the channel power profile vector $\boldsymbol{\alpha}$, channel delay profile vector $\boldsymbol{\beta}$, and subcarrier allocation vector $\mathbf{s}$.

C. Diversity order and Coding Gain

Systematic code design methods which provide maximum coding gain and diversity order for SF codes can guarantee reliable data transmissions in broadband wireless communication systems. Noncoherent and differential space frequency (or space time) codes can potentially achieve the same maximum diversity order TRL as coherent cases over frequency-selective channels by appropriate code construction methods [2] [5] [12]. We can achieve the maximum transmit diversity TL by making the Krylov codeword $\mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G - \mathbf{C}_{G'})$ full-rank (rank TL) for any codeword pair $\mathbf{C}_G$ and $\mathbf{C}_{G'}$ [1]. We want to keep Q as large as possible in order the obtain larger diversity gain. On the other hand, Q decreases if we increase the number of SF codewords transmitted in parallel, N/Q. Therefore, we need to realize a trade off between the diversity order and the code rate especially in channels with long delay spread. Differential SF group codes with $Q \leq TL$ are developed in this paper. The code rate is defined as,

$$\mathcal{R} = \frac{1}{Q}\log_2 M \ \text{ bits/subcarrier use}, \tag{9}$$

where M is the constellation size.

In order to maximize the coding gain, the minimum product of the nonzero eigenvalues of $\mathcal{V}$, which is called the minimum *Krylov product distance* $\Lambda_{\mathcal{K}}$ defined as (cf. [1, Eq. (16)]):

$$\Lambda_{\mathcal{K}} = \min_{\mathbf{C}_G,\mathbf{C}'_G \in \mathcal{C}} \Lambda_{\mathcal{K}}(\mathbf{C}_G, \mathbf{C}_{G'}), \text{ where}$$

$$\Lambda_{\mathcal{K}}(\mathbf{C}_G, \mathbf{C}_{G'}) = \frac{1}{2}\left|\mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G - \mathbf{C}_{G'})^\dagger \cdot \mathbf{K}^L_{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s}}(\mathbf{C}_G - \mathbf{C}_{G'})\right|^{1/TL} \tag{10}$$

has to be maximized.

The channel power profile can be decoupled from the rest of the variables in the Krylov Product distance to obtain the *effective Krylov product distance* $\Lambda^*_{\mathcal{K}}$, which is the product distance of $\mathbf{K}^L_{\boldsymbol{\beta},\mathbf{s}}(\mathbf{D})$ [1, Theorem 2]. Then the *effective Krylov product distance* can be written as (cf. [1, Eq. (21)]):

$$\Lambda^*_{\mathcal{K}} = \min_{\mathbf{G}\in\mathcal{G},\mathbf{G}\neq\mathbf{I}} \frac{1}{2}\left|\mathbf{K}^L_{\boldsymbol{\beta},\mathbf{s}}(\mathbf{D})^\dagger(\mathbf{I}-\mathbf{G})^\dagger(\mathbf{I}-\mathbf{G})\mathbf{K}^L_{\boldsymbol{\beta},\mathbf{s}}(\mathbf{D})\right|^{1/TL}, \tag{11}$$

where

$$\mathbf{K}^L_{\boldsymbol{\beta},\mathbf{s}}(\mathbf{D}) = \begin{bmatrix} \mathbf{A}_s^{\beta_1}\mathbf{d} & \mathbf{A}_s^{\beta_1+Z_1}\mathbf{d} & \cdots & \mathbf{A}_s^{\beta_1+Z_{T-1}}\mathbf{d} \\ \cdots\mathbf{A}_s^{\beta_L}\mathbf{d} & \mathbf{A}_s^{\beta_L+Z_1}\mathbf{d} & \cdots & \mathbf{A}_s^{\beta_L+Z_{T-1}}\mathbf{d} \end{bmatrix}, \tag{12}$$

and $Z = (Z_1, \ldots, Z_{T-1})$ is the transmit delay vector which is chosen according to the delay profile β of the channel.

III. Differential SF Group Codes for MIMO-OFDM

A. DSFC Transmit and Receive Model

Let $\mathcal{C}$ be a space-frequency block code of size S: $\mathcal{C} = \{\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_S\}$, where each $\mathbf{C}_m$ is a $Q \times T$ matrix to be transmitted from Q subcarriers and T transmit antennas with $Q \leq N$ where N is the number of OFDM subcarriers. The (q,t)th element of $\mathbf{c}_q$ is transmitted from subcarrier s_q and antenna t.

For a $\mathbf{C}_k \in \mathcal{C}$ let $\mathbf{C}_k = [\mathbf{c}_0^T, \mathbf{c}_1^T, \cdots, \mathbf{c}_{Q-1}^T]^T$, where $\mathbf{c}_q$ is the length T vector to be transmitted from T antennas on subcarrier s_q. The OFDM modulator applies an N-point IFFT to N consecutive data symbols and then prepends a cyclic prefix (CP) of length $L_{CP} \geq L$ (which is a copy of the last L_{CP} samples of the OFDM symbol) to the parallel-to-serial converted OFDM symbol. The receiver first discards the cyclic prefix, and then applies an N-point FFT to each of the R received signals. After FFT demodulation at the receiver, the signal at the s_qth subcarrier is subject to flat-fading and additive complex white Gaussian noise $\mathbf{w}_q$, which is statistically independent among different receiver antennas and different subcarriers, i.e., $\mathcal{E}(\mathbf{w}_q\mathbf{w}_{q'}^\dagger) = \sigma_n^2\mathbf{I}_R\delta[q-q']$. The $R \times 1$ signal vector $\mathbf{y}_q$ received at R receiver antennas at the subcarrier s_q can be expressed as

$$\mathbf{y}_q = \sqrt{\rho/T}\mathbf{H}(s_q)\mathbf{c}_q + \mathbf{w}_q \quad q = 0, 1, \cdots, Q-1, \tag{13}$$

where ρ is the average SNR at each receive antenna. (13) can be rewritten as

$$\mathbf{Y}_k = \sqrt{\rho/T}\widetilde{\mathbf{C}}_k\mathbf{H}_k + \mathbf{W}_k, \quad k = 1, \cdots, N/Q, \tag{14}$$

where

$$\mathbf{Y_k} = \begin{bmatrix} \mathbf{y}_0^T \\ \mathbf{y}_1^T \\ \vdots \\ \mathbf{y}_{Q-1}^T \end{bmatrix}, \widetilde{\mathbf{C}}_k = \begin{bmatrix} \mathbf{c}_0^T & & & \\ & \mathbf{c}_1^T & & \\ & & \ddots & \\ & & & \mathbf{c}_{Q-1}^T \end{bmatrix}$$
$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}(s_o)^T \\ \mathbf{H}(s_1)^T \\ \vdots \\ \mathbf{H}(s_{Q-1})^T \end{bmatrix}, \mathbf{W_k} = \begin{bmatrix} \mathbf{w}_0^T \\ \mathbf{w}_1^T \\ \vdots \\ \mathbf{w}_{Q-1}^T \end{bmatrix}. \tag{15}$$

Differential space-frequency modulation is designed such that a sequence of messages is differentially encoded in a way similar to differential PSK across the OFDM subcarriers in a group manner for T transmit antennas. We assume that at

$k = 0$, we transmit $\mathbf{C}_0 = \mathbf{D}$ to initialize the transmission. Thereafter, we encode the transmitted signals differentially in the frequency dimension within each OFDM block as follows (cf. [7, Eq. (17)]):

$$\mathbf{C}_k = \mathbf{G}_k \mathbf{C}_{k-1}, \quad k = 1, \ldots, N/Q, \tag{16}$$

where $\mathbf{G}_k$ belongs to a set of $Q \times Q$ unitary generating matrices $\mathcal{G} = \{\mathbf{I}, \mathbf{G}_0, \mathbf{G}_0^2, \cdots, \mathbf{G}_0^{M-1}\}$, where

$$\mathbf{G}_0 = \begin{bmatrix} e^{-j\frac{2\pi}{M}\vartheta_0} & 0 & \cdots & 0 \\ 0 & e^{-j\frac{2\pi}{M}\vartheta_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j\frac{2\pi}{M}\vartheta_{Q-1}} \end{bmatrix} \tag{17}$$

and $\mathbf{G}_k {\mathbf{G}_k}^\dagger = \mathbf{G}_k^\dagger \mathbf{G}_k = \mathbf{I_Q}$, $\mathbf{D}$ is a fixed $Q \times T$ complex matrix called the *initial or starting matrix* [1] [7]. Therefore, the set of all code matrices are defined in $G\mathbf{D}$.

The group property guarantees that $\mathbf{C_k} \in \mathcal{G}\mathbf{D}$ whenever $\mathbf{C_{k-1}} \in \mathcal{G}\mathbf{D}$ [7]. Note that the differential encoding of (16) meets the energy constraint

$$\frac{1}{TQ}\mathrm{Tr}\left(\mathbf{C}_G \mathbf{C}_G^\dagger\right) = 1, \text{ for every } \mathbf{C}_G \in \mathcal{C}. \tag{18}$$

Differential decoding is performed over two consecutive received matrices $\mathbf{Y}_k$ and $\mathbf{Y}_{k-1}$ for $k = 1, \cdots, N/Q$ as follows (see [10])

$$\widehat{\mathbf{G}} = \arg\max_{\mathbf{G}\in\mathcal{G}} \mathrm{ReTr}\{\mathbf{G}\mathbf{Y}_k^\dagger \mathbf{Y}_{k-1}\}. \tag{19}$$

B. Starting signal $\mathbf{D}$ *and subcarrier allocation vector* $\mathbf{s}$

The optimal starting matrix $\mathbf{D}$ and subcarrier allocation vector $\mathbf{s}$ should satisfy the conditions such that the corresponding effective Krylov codeword $\mathbf{K}_{\beta,\mathbf{s}}(\mathbf{D})$ is unitary, i.e. $\mathbf{K}_{\beta,\mathbf{s}}(\mathbf{D})^\dagger \cdot \mathbf{K}_{\beta,\mathbf{s}}(\mathbf{D}) = \mathrm{Q} \cdot \mathbf{I}$ under the transmit power constraint [1, Eq. (22)].

$$\frac{1}{TLQ} Tr\left(\mathbf{K}_{\beta,\mathbf{s}}(\mathbf{D})\mathbf{K}_{\beta,\mathbf{s}}(\mathbf{D})^\dagger\right) = 1. \tag{20}$$

For T transmit antennas, the starting matrix $\mathbf{D}$ can be chosen as [1, Eq. (31)],

$$\mathbf{D}_{[\mathbf{Q}\times\mathbf{T}]} = \begin{bmatrix}\mathbf{d} & \mathbf{A}_s^{Z_1}\mathbf{d} & \mathbf{A}_s^{Z_2}\mathbf{d} & \cdots & \mathbf{A}_s^{Z_{T-1}}\mathbf{d}\end{bmatrix} \\ \rightarrow \textit{space}, \ \downarrow \textit{frequency} \tag{21}$$

where $\mathbf{d}$ is the *starting matrix* [1, Eq. (27)]

$$\mathbf{d}_{[Q\times 1]} = \mathbf{1}_{Q\times 1} = \begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix}, \downarrow \textit{frequency} \tag{22}$$

and $\mathbf{A_s}$ is defined in (8).

The other condition to make the effective Krylov codeword matrix $\mathbf{K}_{\beta,\mathbf{s}}(\mathbf{D})$ unitary is to interleave the subcarrier group for each SF block $\mathbf{s} = (s_0, s_1, \cdots, s_{Q-1})$ across the whole bandwidth as [1, Eq. (29)],

$$s_q = s_0 + q \cdot N/Q, \tag{23}$$

where the (q,t)'th element of $\mathbf{C}_m$ is transmitted from subcarrier s_q and antenna t, and s_0 can take N/Q different values, one for each of the N/Q codewords transmitted in parallel: $s_0 = 0, 1, \ldots, N/Q - 1$.

By choosing $\mathbf{D}$ as in (21), we can obtain non-unitary QxT differential matrices $\mathbf{C}_k$ if $Q \neq T$. However, those non-unitary codewords meet the constant power constraint in (18) due to the design criteria. The energy constraint avoids blowing up the transmit power as well as a vanishing signal during the differential encoding of nonunitary signals, hence a scaling factor as in [13] does not need to be introduced at the transmitter to keep the average transmit energy constant.

IV. Simulation Results

We simulate the proposed differential SF group codes for a system with $T = 2$ transmit and $R = 1$ receive antennas. We consider a MIMO-OFDM system based on the IEEE 802.11a standard [1] [10]. We have non-line of sight (NLOS) Nokia rooftop wideband channel model in a suburban environment. Its root mean-squared (rms) delay spread is about 49 ns, and a 10-tap delay line model is built to model its average power-delay profile [1].

Table I lists the best found parameters $(\vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3)$ and transmit delay vector Z_1 that maximizes the *effective Krylov product distance* $\Lambda_{\mathcal{K}}^*$, defined in (11) through exhaustive computer search. Notice that the coding advantage of differential space frequency group codes is exactly half of the coding advantage of optimal space frequency group codes in [1, Table I].

SF Mapping is a technique which can map whatever full diversity ST code into a full diversity SF code by repeating each ST signals L times over adjacent subcarriers [15]. Since the unitary ST group code is an optimal space-time group code, the mapping technique uses this code as a basis for generating SF code. A SF code is obtained by repeating a 2×2 unitary ST cyclic group code $\mathbf{C} \in \mathcal{G}\mathbf{D}$ twice over adjacent subcarriers [15], where

$$\mathbf{D} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad \mathcal{G} = \{\mathbf{I}, \Theta, \cdots, \Theta^{M-1}\}, \\ \Theta = \begin{bmatrix} e^{2\pi j/M} & 0 \\ 0 & e^{2\pi jk/M} \end{bmatrix} \tag{24}$$

and $k = 1$ for BPSK and QPSK, $k = 2$ for 8-PSK, $k = 7$ for 16-PSK and 32-PSK.

The performance of proposed differential SF group codes is evaluated in terms of coding gain improvement and BER performance. Table I and Table II show the coding advantage of the differential SF code is higher than the coding gain of a SF mapping code in a Nokia rooftop channel scenario for all modulation scenarios. Coding gain improves BER performance of differential space frequency codes especially at low SNRs.

In Fig. 1, simulation results show that the differential SF group codes based on optimum coherent SF group codes proposed in [1] perform better than optimum mapping technique proposed in [15] using 16-PSK modulation (which

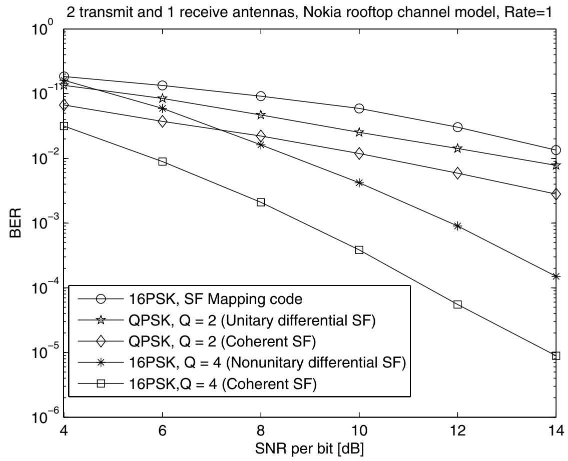

Fig. 1. Performance comparison of differential SF code, optimal coherent SF codes and SF mapping code in MIMO-OFDM system for Nokia rooftop channel when $\mathcal{R} = 1$.

TABLE I
CODING ADVANTAGE OF DIFFERENTIAL SPACE-FREQUENCY GROUP CODES (N=64, Q=4, T=2)

	$(M;\,\vartheta_0,\vartheta_1,\vartheta_2,\vartheta_3)$	$(Z_1;\Lambda^*_{\mathcal{K}})$
BPSK	(2; 1, 1, 1, 1)	(2; 2.985)
QPSK	(4; 1, 1, 1, 1)	(2; 1.49)
8PSK	(8; 1, 1, 3, 5)	(2; 1.055)
16PSK	(16; 1, 3, 5, 7)	(2; 0.885)
32PSK	(32; 1, 9, 17, 25)	(2; 0.435)

corresponds to $\mathcal{R} = 1$) when $Q = 4$, i.e. the diversity order is 4. It also shows that coherent detection outperforms the corresponding differential detection by 3 dB. On the other hand, implementing coherent detectors for optimal space frequency codes and SF mapping techniques require robust channel estimation algorithms. Fig. 1 also depicts the performance results of QPSK nonunitary ($Q = 4$, $T = 2$) and 16-PSK unitary ($Q = 2$, $T = 2$) differential SF group codes in Nokia rooftop channel model with optimal QPSK and 16-PSK coherent SF codes respectively. The reason behind using different modulation types for unitary and nonunitary DSFCs is to make fair comparisons by assigning same code rates $\mathcal{R} = 1$ to each code construction methods. Fig. 1 shows that the performance of nonunitary code with $Q = 4$ gives better BER improvements compared with the unitary DSFCs with $Q = 2$ due to the high exploitation of diversity order Q especially in high SNR. Due to the energy restriction on code construction method, the repeated multiplication in differential encoding of non-unitary codewords does not cause the codewords to blow up or diminish.

V. CONCLUSIONS

In this paper, DSFC, a novel differential encoding and decoding scheme across antennas and OFDM tones (space-frequency coding) within each single OFDM symbol is proposed. DSFCs are robust against scenarios where the fading channel may change from one OFDM symbol to another

TABLE II
CODING ADVANTAGE OF SPACE-FREQUENCY MAPPING CODE [15] (N=64, Q=4, T=2)

$\mathcal{A}^+$	$(M;\,\vartheta_0,\vartheta_1,\vartheta_2,\vartheta_3)$	$(Z_1;\Lambda^*_{\mathcal{K}})$
BPSK	(2; 1)	(1; 0.58)
QPSK	(4; 1, 3)	(1; 0.29)
8PSK	(8; 3, 5)	(1; 0.21)
16PSK	(16; 7, 9)	(1; 0.09)
32PSK	(32; 7, 9, 23, 25)	(1; 0.04)

independently. Based on the code construction method, the power profile of the channel decouples from the choice of codewords in the design criteria, hence the transmitter needs only the knowledge of delay profile of the channel. The performance of nonunitary and unitary DSFCs is investigated in frequency-selective channels with multiple taps. Nonunitary DSFCs with $Q = 4$ gives better performance results than unitary DSFCs codes with $Q = 2$ when $\mathcal{R} = 1$ especially at high SNRs due to higher exploitation of diversity order.

For future work, the performance of DSFCs can be investigated for differential encoding over partial subcarriers for MIMO-OFDM. This technique can be applied when there are severe variations in the spectrum of the channel impulse responses, “bad” part of the channel's subcarriers is not modulated to increase the overall performance of the system.

APPENDIX

PROOF OF THEOREM

The derivation of Chernoff upper bound has been presented previously in [7] [14]. In this paper, since the SF signal is coded on a subcarrier group $(s_0, s_1, \cdots, s_{Q-1})$, instead on all the N subcarrier of an OFDM symbol, the Chernoff upper bound on the PEP for erroneously decoding the transmitted signal $\mathbf{C}_G$ as $\mathbf{C}_{G'}$ is presented below which is similar to [16, pp. 27-30].

The PEP for erroneously decoding C_0 when C_1 is actually transmitted for differential receivers is [7, Eq. (19)],

$$P(\mathbf{C}_0 \to \mathbf{C}_1) \le \prod_{r=1}^{R} \prod_{i=1}^{rank(\mathcal{Z})} \frac{1}{1+\frac{T\rho^2}{4(1+2T\rho)}\lambda_i(\mathcal{Z})}. \qquad (26)$$

The Chernoff upper bound on the PEP for erroneously decoding the transmitted signal C_G as $C_{G'}$ is [7],

$$P\left(\mathbf{C}_G \to \mathbf{C}_{G'}|\mathbf{H}\right) \le e^{-\frac{T\rho^2}{4(1+2T\rho)}||\mathbf{X}||_F^2}, \qquad (27)$$

where $||\mathbf{X}||_F^2 = \sum_{q=0}^{Q-1}||\mathbf{H}(s_q)\mathbf{x}_q||_F^2$, $\mathbf{x}_q = \mathbf{c}_q - \mathbf{e}_q$.

Averaging the PEP over the random channels,

$$P\left(\mathbf{C} \to \mathbf{E}\right) = \mathcal{E}\left\{P\left(\mathbf{C} \to \mathbf{E}|\mathbf{H}\right)\right\} \le \prod_{i=1}^{r(\mathcal{V}_\mathbf{X})} \frac{1}{1+\frac{T\rho^2}{2(1+2T\rho)}\lambda_i(\mathcal{V}_\mathbf{X})}. \qquad (28)$$

$\mathcal{V}_\mathbf{X}$ is the covariance matrix of $\mathbf{X}$ and can be expressed as:

$$\mathcal{V}_\mathbf{X} = \frac{1}{2}\mathcal{E}\{\mathbf{X}\mathbf{X}^\dagger\}, \qquad (29)$$

$$\mathcal{V} = \sum_{l=1}^{L}\alpha_l^2\begin{bmatrix} \bar{\beta}_l^0\mathbf{x}_0^\dagger\mathbf{x}_0 & \bar{\beta}_l^{s_1-s_0}\mathbf{x}_0^\dagger\mathbf{x}_1 & \cdots & \cdots & \bar{\beta}_l^{s_{Q-1}-s_0}\mathbf{x}_0^\dagger\mathbf{x}_{s_{Q-1}} \\ \bar{\beta}_l^{s_0-s_1}\mathbf{x}_1^\dagger\mathbf{x}_0 & \bar{\beta}_l^0\mathbf{x}_1^\dagger\mathbf{x}_1 & \cdots & \cdots & \bar{\beta}_l^{s_{Q-1}-s_1}\mathbf{x}_1^\dagger\mathbf{x}_{Q-1} \\ \bar{\beta}_l^{s_0-s_2}\mathbf{x}_2^\dagger\mathbf{x}_0 & \bar{\beta}_l^{s_1-s_2}\mathbf{x}_2^\dagger\mathbf{x}_1 & \ddots & \vdots & \bar{\beta}_l^{s_{Q-1}-s_2}\mathbf{x}_2^\dagger\mathbf{x}_{Q-1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \bar{\beta}_l^{s_0-s_{Q-1}}\mathbf{x}_{Q-1}^\dagger\mathbf{x}_0 & \bar{\beta}_l^{s_1-s_{Q-1}}\mathbf{x}_{Q-1}^\dagger\mathbf{x}_1 & \cdots & \cdots & \bar{\beta}_l^0\mathbf{x}_{Q-1}^\dagger\mathbf{x}_{Q-1} \end{bmatrix}$$

$$= \sum_{l=1}^{L}\alpha_l^2\mathbf{A}_s^{\beta_l}(\mathbf{C}-\mathbf{E})(\mathbf{C}-\mathbf{E})^\dagger\mathbf{A}_s^{-\beta_l}$$

$$= \mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}-\mathbf{E})\cdot\mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}-\mathbf{E})^\dagger \quad (25)$$

where each entry pair (q, q') $(q, q' = 0, 1, \cdots, Q-1)$ in the covariance matrix has a form

$$\mathcal{E}\quad\{\mathbf{H}(s_q)\mathbf{x}_q\mathbf{x}_{q'}^\dagger\mathbf{H}(s_{q'})^\dagger\}$$

$$= \mathcal{E}\quad\left\{\begin{bmatrix}\mathbf{H}(s_q^1)\\ \vdots \\ \mathbf{H}(s_q^R)\end{bmatrix}\mathbf{x}_q\mathbf{x}_{q'}^\dagger\begin{bmatrix}\mathbf{H}(s_{q'}^1)^\dagger & \cdots & \mathbf{H}(s_{q'}^R)^\dagger\end{bmatrix}\right\}.$$

$\mathbf{H}(s_q^r)$ denotes a $1 \times T$ row vector, which is the frequency response of the channel from T transmitters to the r-th receiver for the s_q-th subcarrier. If we assume the receiver-antenna gain responses are uncorrelated, the above matrix will be diagonal and the diagonal elements are

$$\mathcal{E}\left\{\mathbf{H}(s_q^r)\mathbf{x}_q\mathbf{x}_{q'}^\dagger\mathbf{H}(s_{q'}^r)^\dagger\right\} = \mathrm{Tr}\left\{\mathbf{x}_{q'}^\dagger\mathcal{E}\left[\mathbf{H}(s_{q'}^r)^\dagger\mathbf{H}(s_q^r)\right]\mathbf{x}_q\right\}, \quad (30)$$

for $r = 1, 2, \cdots, R$.

$$\mathcal{E}\{\mathbf{H}(s_{q'}^r)^\dagger\mathbf{H}(s_q^r)\} = \sum_{l=1}^{L}\alpha_l^2 e^{j\frac{2\pi}{N}(s_{q'}-s_q)\beta_l}\mathbf{I}_T. \quad (31)$$

Substituting (31) and (30) into (30),

$$\mathcal{E}\{\mathbf{H}(s_q)\mathbf{x}_q\mathbf{x}_{q'}^\dagger\mathbf{H}(s_{q'})^\dagger\} = \mathbf{x}_{q'}^\dagger\mathbf{x}_q\left(\sum_{l=1}^{L}\alpha_l^2 e^{j\frac{2\pi}{N}(s_{q'}-s_q)\beta_l}\right)\mathbf{I}_R, \quad (32)$$

Next (32) is substituted into the covariance matrix expression (29) to derive

$$\mathcal{V}_{\mathbf{X}} = \frac{1}{2}\mathcal{V}\otimes\mathbf{I}_R, \quad (33)$$

where $\otimes$ denotes tensor product. $\mathcal{V}$ is given at the top of the page where $\bar{\beta}_l = e^{j\frac{2\pi}{N}\beta_l}$ in Eq. (25).

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