CHANNEL PHASE AND DATA ESTIMATION IN SLOWLY FADING FREQUENCY NONSELECTIVE CHANNELS

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M.Sc. Thesis Presentation

OUTLINE

1) Introduction
2) Multipath Fading Channel Characterization
3) Optimum Decoding Based Smoothing Algorithm (ODSA)
4) Optimum Receivers for BOFSK and BPSK in Slowly fading frequency-nonselective channels
5) Simulation Results
6) Conclusion & Future Work

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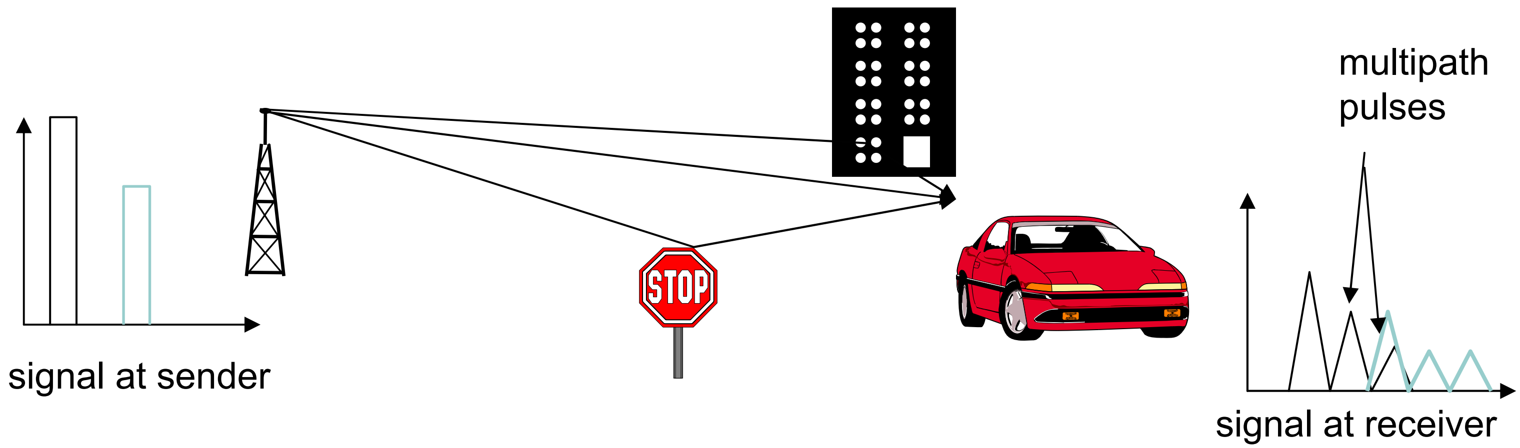
Multipath Propagation



Signal can take many different paths between sender and receiver due to reflection, scattering, diffraction

When signals arrive at the receiving antenna having traversed different paths, they may combine destructively. This, *multipath*, phenomenon can induce signal *fading*

Multipath Propagation

- Fading channels are multiplicative-noise channels and result in bursts of errors, as, for example, in

  1) Wireless communication

  2) Compact-disc players
- The multiplicative nature of the channel means increasing signal power may not yield a proportional improvement in performance

Modelling of Radio Channel

- Characterization of multipath medium is useful in designing the transmitter and receiver that can reduce the effect of multipath fading.
- Channel estimation is essential to mitigate the effects of multipath fading at the receiver.

Literature Survey (1/5)

- Considerable amount of research is devoted to channel estimation in Flat Fading Channels.
- Optimal MLSE methods in fading channels are studied. Implementation is difficult.
- Requires high dimensional filtering.

Literature Survey (2/5)

- MLSE produces hard decisions and is unsuitable for applications requiring soft decisions.
- Hence, joint demodulation and decoding in a single stage is not practical due to required number of stages in concatenated processing structures (e.g. modems using error correction codes with interleaving).

Literature Survey (3/5)

- Most of literature estimates the complex Gaussian channel coefficient by employing linear estimators or Kalman filters in Flat Fading Channels.
- Two stage implementable but suboptimal receivers are commonly utilized for MLSE.

Literature Survey (4/5)

- Linear estimators or Kalman Filters are employed and estimation is simplified by per-survivor processing, pilot symbol assisted modulation, both pilot symbols and tentative data decisions and expected Maximization (EM) Algorithm .
- Decision Feedback Adaptive Linear Predicition (DFALP) to track phase and amplitude is proposed.

Literature Survey (5/5)

- Carrier Phase Tracking Loops (like PLL) are not optimum phase estimators, fading causes significant degradation degradation to phase tracking performance of PLL
- Advantage of estimating complex Gaussian coefficient is estimating both amplitude and phase of the channel.

Motivation (1/2)

- Channel Estimation is essential for coherent detection.
- We need to estimate channel at every symbol duration due to the variability of the channel.
- Robustness & Computational complexities are important.
- Objective: Estimate channel parameters for coherent modulation schemes.

Motivation (2/2)

- In this study, Channel Phase estimation in a transmitted sequence is investigated.
- Then, the data estimation is obtained in a symbol-by-symbol receiver using the estimated channel phase in a transmitted sequence.

Constant Envelope Modulation

Constant envelope modulation: Modulation methods whose signal has constant envelope→Information modulated in carrier frequency.

Advantages:

1-) Immune against signal fluctuations due to fading.

2-) Power efficient amplifiers can be used

Why Phase Estimation?

- In PSK & FSK, phase estimation is more significant compared to amplitude estimation.
- Simple estimation of amplitude and powerful estimation of phase performs nearly same as perfect estimates.
- ODSA is used for MAP channel phase estimation that appears nonlinearly.

Why ODSA?

- In general, nonlinear estimators are complex & difficult to realize.
- Suboptimum algorithms like Extended Kalman Filters (EKF) can be proposed.
- EKF can sometimes cause suboptimal and biased estimates with no reliable accuracy.
- ODSA ensures reliable guarantees on performance, accuracy, convergence and optimal estimation

Why Symbol-By Symbol Receiver?

- Fading channels are not *memoryless* and correlated process. Thus, sequence estimation is required for optimum detection
- To ease implementation complexity, symbol-by-symbol detection is desirable.

Often Used Channel Models

1) **Rayleigh Fading:** the signal amplitude is Rayleigh distributed (no direct specular component is present, only diffused components)
2) **Ricean Fading :** Besides the diffuse component, there is specular (line-of-sight) component

Rayleigh Fading Channels

- Experimental and theoretically modeled $p(\alpha)$

$$p(\alpha) = \frac{\alpha}{\sigma_1^2} \exp\left[-\frac{\alpha^2}{2\sigma_1^2} \right]$$

Coherent & Noncoherent Detection in Multipath Fading Channels

- Coherent detection: Channel Phase is known at the receiver
- Noncoherent detection: Noncoherent detection with no assumption of channel estimation can be used for fast fading channel (mobile system or high frequency carrier) where channel parameters can not be estimated accurately.

Why Coherent Demodulation in MFC? (1/2)

- Suboptimal and power efficient ways may be used (noncoherent or differentially coherent techniques)
- These techniques have irreducible error floor at high rates, power inefficient, require grater bandwidth like in pilot symbol approaches.

Why Coherent Demodulation in MFC? (2/2)

- Coherent demodulation has advantage in SNR compared to noncoherent demodulation
- Coherent demodulation has superior performance at the expense of increase in computational complexity.

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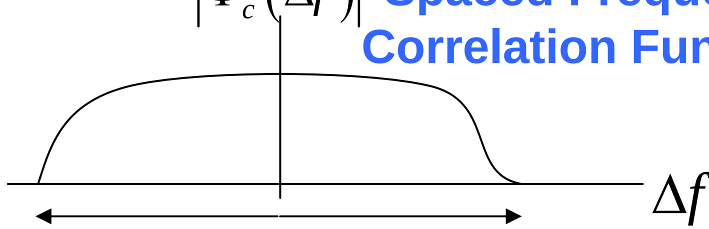
Channel correlation and Power density functions

Multipath Intensity Profile

T_m = multipath spread

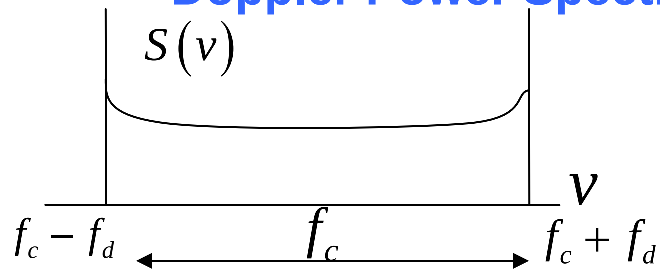
Fourier Transforms

$|\Phi_c(\Delta f)|$ **Spaced Frequency Correlation Function**



$(\Delta f)_c \approx \frac{1}{T_m}$ Coherence bandwidth

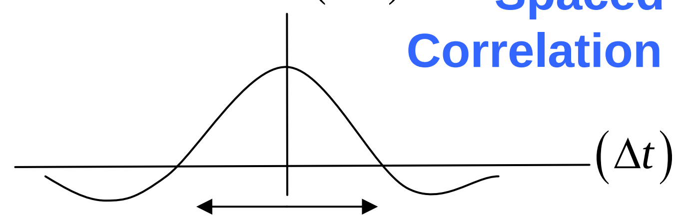
Doppler Power Spectrum



f_d = Spectral Broading

Fourier Transforms

$\Phi_c'(\Delta t)$ **Spaced Time Correlation Function**



$(\Delta t)_c \approx \frac{1}{f_d}$ Coherence Time

SMALL SCALE FADING CHARACTERIZATION OF MULTIPATH FADING CHANNELS

Time-delay Domain

- Frequency- selective fading(ISI distortion, pulse mutilation, irreducible BER)

  Multipath delay spread > symbol time
- Flat fading(loss in SNR)
  Multipath delay spread < symbol time

Dual Mechanism

Doppler Shift Domain

- Fast fading(high Doppler, PLL failure, irreducible BER)
  Channel fading rate > symbol rate
- Slow fading(low Doppler, loss in SNR)
  Channel fading rate < symbol rate

Frequency Domain

- Frequency- selective fading(ISI distortion, pulse mutilation, irreducible BER)

  Channel coherence BW < symbol rate
- Flat fading(loss in SNR)

  Channel coherence BW > symbol rate

Dual Mechanism

Time Domain

- Fast fading(high Doppler, loss in SNR)
  Channel coherence time < symbol time
- Slow fading(low Doppler, loss in SNR)

  Channel coherence time > symbol time

Frequency Selective and Frequency Nonselective Channels

Frequency-Selective: The effects of the channel on the information signal are frequency-dependent

In time domain: $T_m > T_s$

In frequency domain: $(\Delta f)_c < 1/T_s$

Frequency-NonSelective:

In time domain: $T_m < T_s$

In frequency domain: $(\Delta f)_c > 1/T_s$

Slow and Fast Fading Channels

***Slow Fading Channels*:**

In time domain: $(\Delta t)_c > T_s$

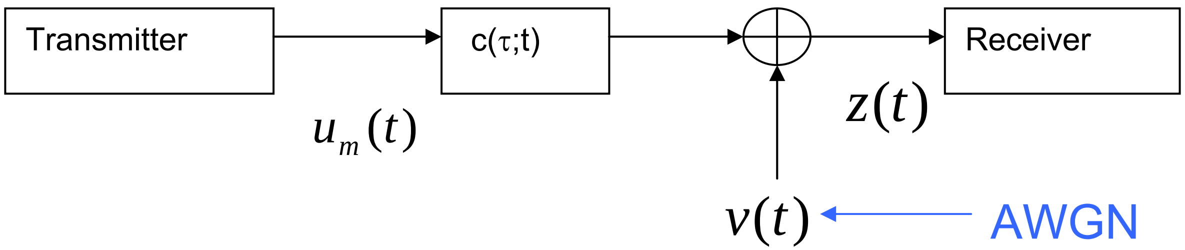
In frequency domain: $W < f_d$

Fast Fading Channels :

In time domain: $(\Delta t)_c < T_s$

In frequency domain: $W > f_d$

SLOWLY FADING AND FREQUENCY NONSELECTIVE CHANNEL MODEL AND ASSUMPTIONS (1/4)



$$z(t) = C(\tau;t) * u_m(t) + v(t)$$

$$\Rightarrow \quad z(t) = \int_{-\infty}^{\infty} C(\tau;t) u_m(t-\tau) d\tau + v(t)$$

$$\Rightarrow \quad C(f;t) = F\{C(\tau;t)\} = \int_{-\infty}^{\infty} C(\tau;t) \exp(-j2\pi f t) d\tau$$

SLOWLY FADING AND FREQUENCY NONSELECTIVE CHANNEL MODEL AND ASSUMPTIONS (2/4)

$$z(t) = \int_{-\infty}^{\infty} C(f;t) U_m(f) \exp(j2\pi ft) d\tau + v(t)$$

Frequency Nonselective Channel Assumption:

$$z(t) = C(0;t) \int_{-\infty}^{\infty} U_m(f) \exp(j2\pi ft) d\tau + v(t)$$

$$z(t) = C(0;t) u_m(t) + v(t) = \alpha(t) \exp(-j\phi(t)) u_m(t) + v(t)$$

$$C(0;t) = C(t) = X(t) + jY(t) \qquad \alpha(t) = \sqrt{X^2(t) + Y^2(t)} \qquad \phi(t) = \tan^{-1}\left(\frac{Y(t)}{X(t)}\right)$$

SLOWLY FADING AND FREQUENCY NONSELECTIVE CHANNEL MODEL AND ASSUMPTIONS (3/4)

Slowly Fading Assumption: the time-variant complex channel coefficient changes slowly (=> constant during a symbol interval)

$$z(t) = \alpha(t)\exp\left(-j\phi(t)\right)u_m(t) + v(t)$$

$$\alpha(t) \approx \alpha$$

Constant in one symbol interval, α is Rayleigh distributed

$$\phi(t) = \phi$$

Constant in one symbol interval, ϕ is uniformly distributed between $(0,2\pi)$

SLOWLY FADING AND FREQUENCY NONSELECTIVE CHANNEL MODEL AND ASSUMPTIONS (4/4)

$$z(t) = c(t)u_m(t) + v(t)$$

$C(t)$ is approximated by piece-wise constant process $C(k)$

$$C(k) = X(k) + jY(k) = \alpha(k)\exp(-j\phi(k))$$

$$X(k+1) = \lambda X(k) + n_1(k) \qquad n_1(k) : N\left(0, \gamma\left(1-\lambda^2\right)\right)$$

$$Y(k+1) = \lambda Y(k) + n_2(k) \qquad n_2(k) : N\left(0, \gamma\left(1-\lambda^2\right)\right)$$

$$X(1) : N(0,\gamma) \qquad Y(1) : N(0,\gamma)$$

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OPTIMUM DECODING BASED SMOOTHING ALGORITHM (ODSA)

Models and assumptions:

Phase Transition Model: $\phi(k+1) = f\left(k, \phi(k), w(k)\right)$

Observation Model: $z(k) = g\left(k, \phi(k), \alpha(k), v(k)\right)$

$\phi(0)$ is uniformly distributed between $(0,2\pi)$

$\phi(k)$ is channel phase at symbol k

w(k) is disturbance Gausssian noise vector

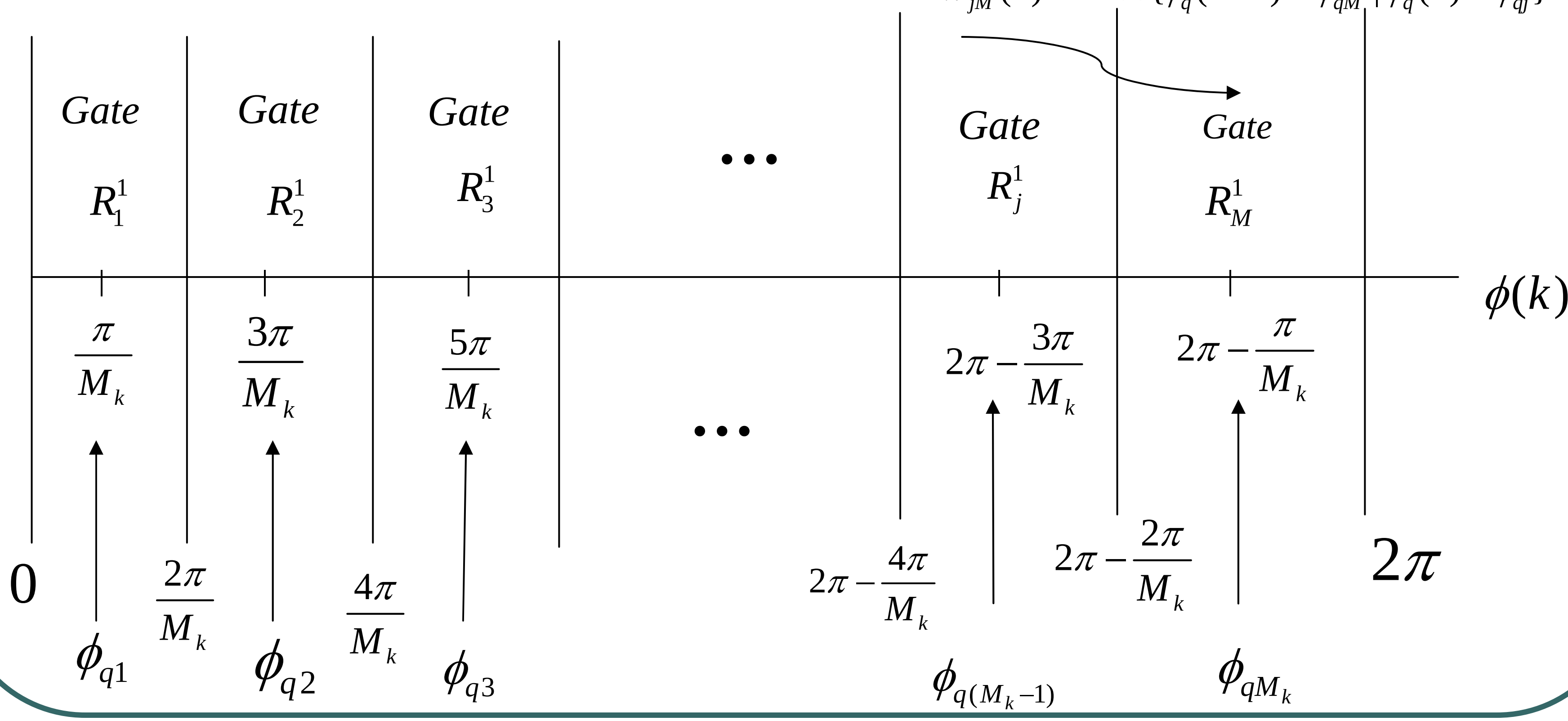
v(k) is observation noise vector

$\alpha(k)$ is interference vector with known statistics

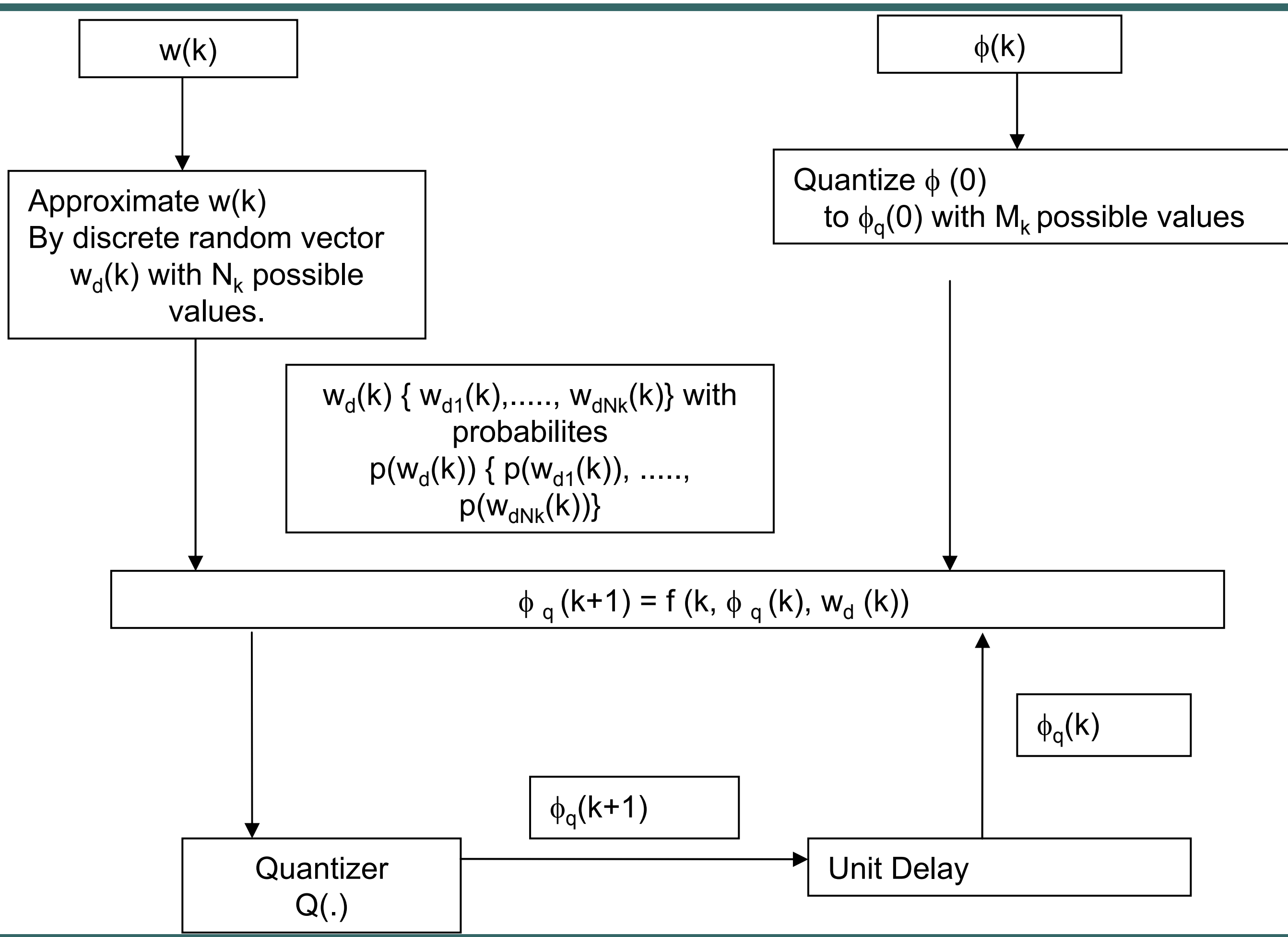
QUANTIZATION OF CHANNEL PHASE

$$\phi_q(k+1) = Q\left\{ f\left(k, \phi_q(k), w_d(k)\right)\right\}$$

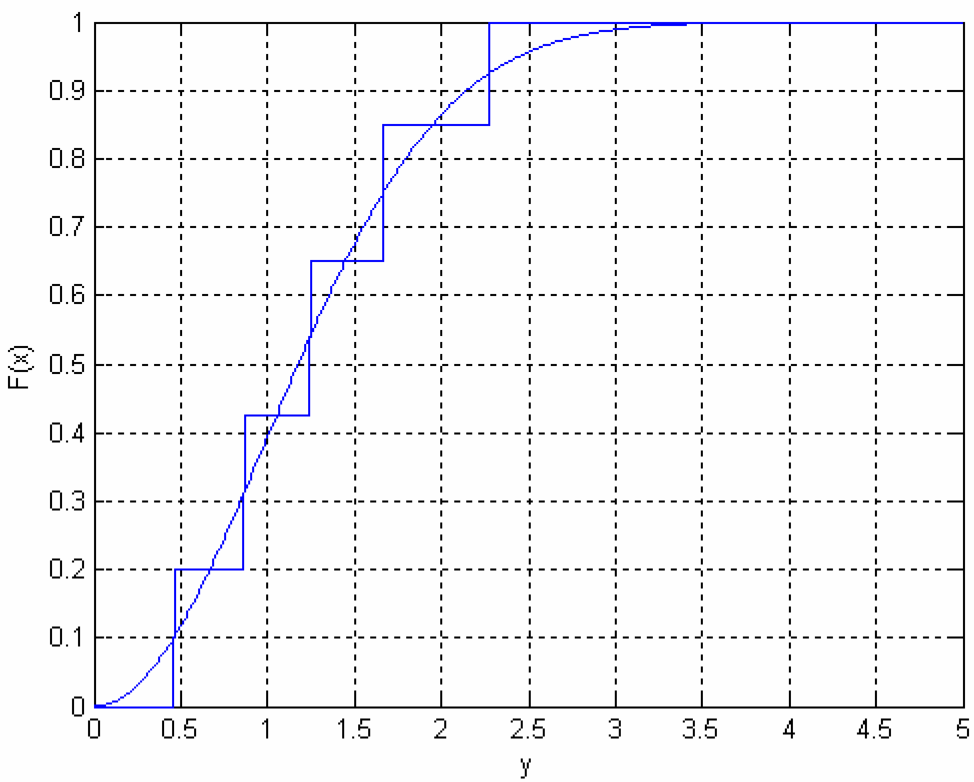
$$\pi_{jM}(k) = \Pr ob\{\phi_q(k+1) = \phi_{qM} \mid \phi_q(k) = \phi_{qj}\}$$



FINITE-STATE MODEL (FSM) FOR THE PHASE TRANSITION MODEL



APPROXIMATION OF AN ABSOLUTELY CONTINUOUS RANDOM VECTOR BY A DISCRETE RANDOM VECTOR (1/3)



APPROXIMATION OF AN ABSOLUTELY CONTINUOUS RANDOM VECTOR BY A DISCRETE RANDOM VECTOR (2/3)

$$J(g(.)) = \int_{-\infty}^{y_1} F_x^{\,2}(a)da + \int_{y_1}^{y_2} [F_x(a) - p_1]^2 da$$

$$+ \int_{y_2}^{y_3} [F_x(a) - (p_1 + p_2)]^2 da + \dots\dots\dots\dots\dots + \int_{y_{n-1}}^{y_n} [F_x(a) - (p_1 + p_2 + \dots\dots + p_{n-1})]^2 da$$

$$+ \int_{y_n}^{\infty} [F_x(a) - 1]^2 da$$

APPROXIMATION OF AN ABSOLUTELY CONTINUOUS RANDOM VECTOR BY A DISCRETE RANDOM VECTOR (3/3)

$$\frac{p_{1,0}}{2} = F_x(y_{1,0}) \qquad (p_{1,0})(y_{2,0} - y_{1,0}) = \int_{y_{1,0}}^{y_{2,0}} F_x(a)da,$$

$$p_{1,0} + \frac{p_{2,0}}{2} = F_x(y_{2,0}) \qquad (p_{1,0}+p_{2,0})(y_{3,0} - y_{2,0}) = \int_{y_{2,0}}^{y_{3,0}} F_x(a)da,$$

$$p_{1,0} + p_{2,0} + \frac{p_{3,0}}{2} = F_x(y_{3,0}) \qquad (p_{1,0}+p_{2,0}+p_{3,0})(y_{4,0} - y_{3,0}) = \int_{y_{3,0}}^{y_{4,0}} F_x(a)da,$$

. .

. .

$$p_{1,0} + p_{2,0} + p_{3,0} + ... + \frac{p_{n,0}}{2} = F_x(y_{n,0}) \qquad (p_{1,0}+p_{2,0}+p_{3,0}+.......+p_{n-1,0})(y_{n,0} - y_{n-1,0}) = \int_{y_{n-1,0}}^{y_{n,0}} F_x(a)da,$$

Transition Probabilities

Transition from R_j to R_M is $\pi_{jM}(k) = \Pr ob\{\phi_q(k+1) = \phi_{qM} \mid \phi_q(k) = \phi_{qj}\}$

The transition is determined by function $Q\{f(k, \phi_q(k) = \phi_{qj}, w_d(k))\}$

If $f(k, \phi_q(k) = \phi_{qj}, w_d(k))$ maps R_j into another gate R_M

for only one possible of $w_{dh}(k)$ Then, $\pi_{jh}(k) = p(w_{dh}(k))$

If $f(k, \phi_q(k) = \phi_{qj}, w_d(k))$ maps R_j into another gate R_M

for two possible of $w_{d1}(k)$ and $w_{d2}(k)$ Then,

$$\pi_{jM}(k) = p(w_{d1}(k)) + p(w_{d2}(k))$$

Transition Probabilities

$$\pi_{jM}(k) = \frac{\Pr ob\left\{\phi(k+1) \in R_m^1, \phi(k) \in R_j^1\right\}}{\Pr ob\left\{\phi(k) \in R_j^1\right\}}$$

$$= \left[\int_{R_j^1} p(\phi(k))\, d\phi(k)\right]^{-1} \left[\int_{R_j^1}\int_{R_m^1} \left[p(\phi(k+1) \mid \phi(k))\, d\phi(k+1)\right] p(\phi(k))\, d\phi(k)\right]$$

$$= \left[\int_{R_j^1} p(\phi(k))\, d\phi(k)\right]^{-1} \left[\int_{R_j^1}\int_{R_m^1} p(\phi(k+1), \phi(k))\, d\phi(k+1)\, d\phi(k)\right]$$

Approximate Observation Model

Apply FSM to Observation Model:

Observation Model: $z(k) = g\left(k, \phi_q(k), \alpha(k), v(k)\right)$

TRELLIS DIAGRAM FOR THE PHASE TRANSITION MODEL (1/2)

Quantized phase vector $\phi_q(k)$ has M_k possible values $\phi_{q1}(k)$, $\phi_{q2}(k)$,...................., $\phi_{qM_k}(k)$ for k=0,.........,L

- We want to represent every possible value of the quantized

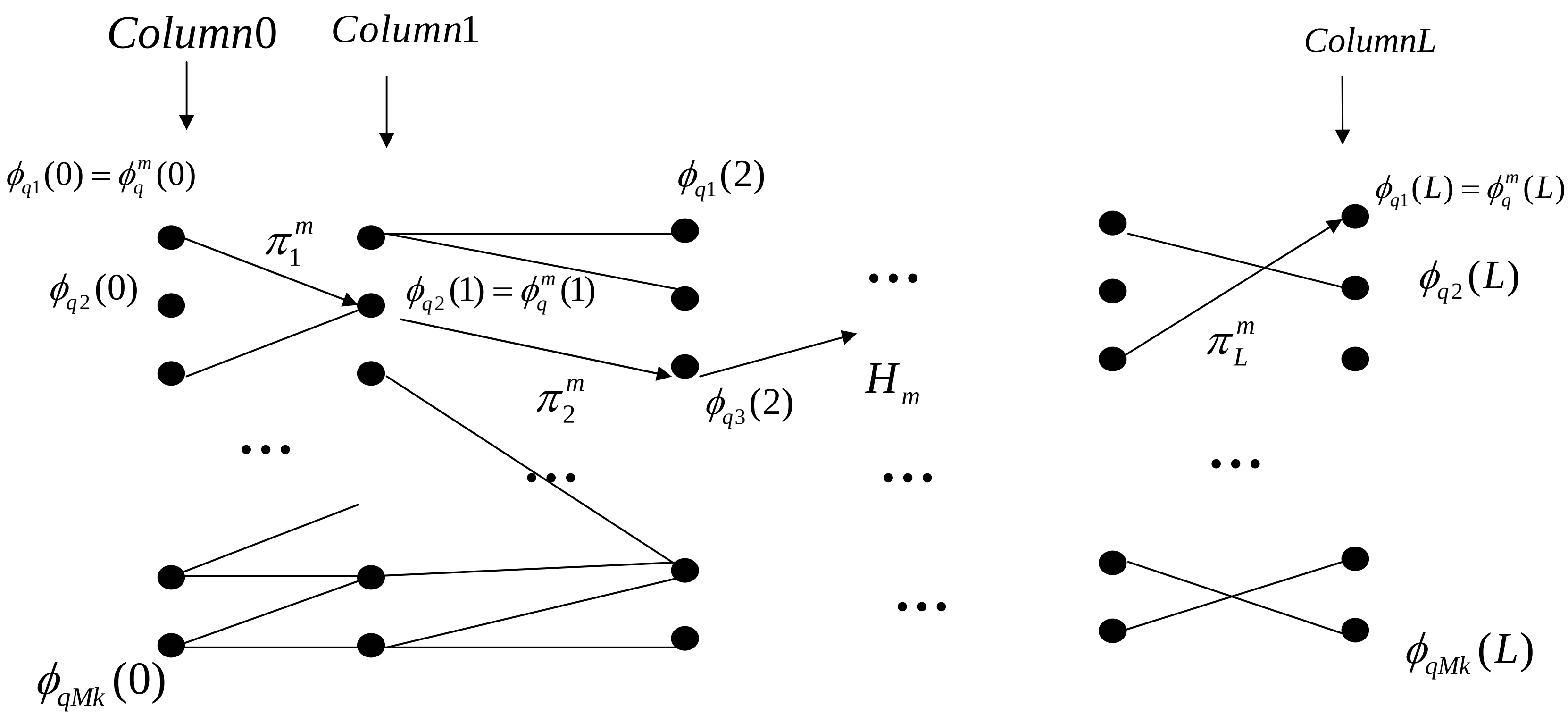
•phase vector $\phi_q(k)$ of the k'th column by points called nodes

- We want to represent the phase transition from one node at the

k'th column to another node at the (k+1)'th column by a line which

indicates the direction of phase transition

TRELLIS DIAGRAM FOR THE PHASE TRANSITION MODEL (2/2)



Notations

M_k	Number of Quantization levels
H_m	The m’th path or Hypothesis
$\phi_q^{\ m}(k)$	The Quantization levels at symbol k which m’th Hypothesis passes
$\pi_0^{\ m}$	Probability at symbol zero at which m’th Hypothesis passes
$\pi_i^{\ m}$	Probability at symbol i at which m’th Hypothesis passes
$\phi_L^{\ m}$	$\left\{\phi_q^m(0), \phi_q^m(1), \phi_q^m(2), ..., \phi_q^m(L)\right\}$
Np	The number of possible paths through the trellis

Notations

$Z^L = \{z(1), \ldots\ldots\ldots\ldots, z(L)\}$: Observation sequence from symbol 0 to L

$\alpha^L = \{\alpha(1), \ldots\ldots\ldots\ldots, \alpha(L)\}$: Interference sequence from symbol 0 to L

Minimum Error Probability Criterion

Minimize the overall probability of error →

$$R = \sum_{j=1}^{Np} \sum_{\substack{i=1 \\ i \neq j}}^{Np} \left\{ \int_{z^L \in D_i} p(H_j) p(Z^L \mid H_j) dz^L \right\}$$

The Optimum Decision Rule is:

Choose H_i if $p(H_i) p(Z^L \mid H_i) > p(H_j) p(Z^L \mid H_j) \quad for \quad all \quad i \neq j$

where $p(Z^L \mid H_j) = \int_{\alpha^L} p(Z^L \mid H_j, \alpha^L) p(\alpha^L) d\alpha^L$

Optimum Decision Rule for the Channel Phase Path (1/5)

$$p(H_i) = \prod_{k=0}^{L} \pi_k{}^i \qquad p(\alpha^L) = \prod_{k=1}^{L} p(\alpha(k))$$

Hence; $p(Z^L \mid H_i) = p(Z^L \mid \phi_L{}^i) = \prod_{k=1}^{L} p(z(k) \mid \phi_q^i(k))$

where $p(z(k) \mid \phi_q^i(k)) = \int_{\alpha(k)} p(z(k) \mid \phi_q^i(k), \alpha(k))\, p(\alpha(k))\, d\alpha(k)$

Approximation:

$$\int_{\alpha(k)} p\left(z(k) \mid \phi_q^i(k), \alpha(k)\right) p\left(\alpha(k)\right) d\alpha(k) \cong$$

$$\sum_{l=1}^{R_k} p\left(z(k) \mid \phi_q^i(k), \alpha_{dl}(k)\right) p\left(\alpha_{dl}(k)\right)$$

R_k is the number of possible values of approximating discrete vector $\alpha_d(k)$

Optimum Decision Rule for the Channel Phase Path (2/5)

Therefore, we make another approximation to observation model:

$$z(k) = g\left(k, \phi_q(k), \alpha(k) = \alpha_d(k), v(k)\right)$$

The Optimum Decision Rule becomes:

$$Choose \quad H_i \quad if \quad \pi_0^i \prod_{k=1}^{L} \pi_k^i p\left(z(k) \mid \phi_q^i(k)\right) > \pi_0^j \prod_{k=1}^{L} \pi_k^j p\left(z(k) \mid \phi_q^j(k)\right)$$

$$for \quad all \quad j \neq i$$

Optimum Decision Rule for the Channel Phase Path (3/5)

Or equivalently;

$$\text{Choose} \quad H_i \quad \text{if} \quad \ln\left(\pi_0^i\right)+\sum_{k=1}^{L}\left\{\ln\left(\pi_k^i\right)+\ln\left(p\left(z(k)\,|\,\phi_q^i(k)\right)\right)\right\} \quad \text{for} \quad \text{all} \quad j\neq i$$

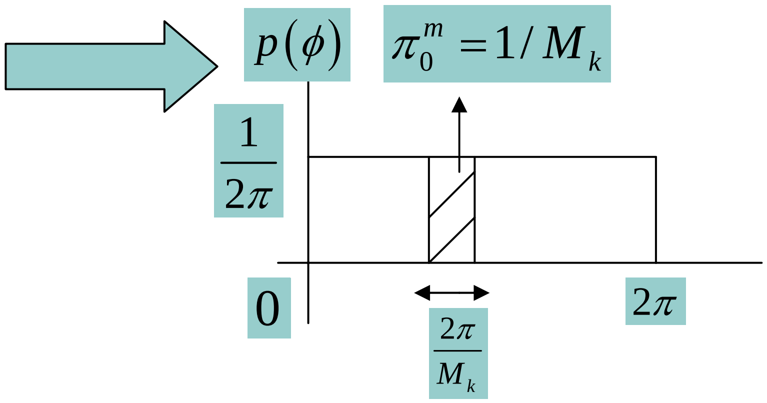
$$>\ln\left(\pi_0^j\right)+\sum_{k=1}^{L}\left\{\ln\left(\pi_k^j\right)+\ln\left(p\left(z(k)\,|\,\phi_q^j(k)\right)\right)\right\}$$

where $p\left(z(k)\,|\,\phi_q^i(k)\right)=\sum_{l=1}^{R_k} p\left(z(k)\,|\,\phi_q^i(k),\alpha_{dl}(k))\,p(\alpha_{dl}(k)\right)$

and $p\left(z(k)\,|\,\phi_q^i(k),\alpha_{dl}(k)\right)=p\left(z(k)\,|\,\phi_q(k)=\phi_q^i(k),\alpha_d(k)=\alpha_{dl}(k)\right)$

Optimum Decision Rule for the Channel Phase Path (4/5)

Definition 1: $MN\left\{\phi_{qi}(0)\right\} = \ln\left\{\Pr ob\left(\phi_q(0) = \phi_{qi}(0)\right)\right\}$



$$MN\left\{\phi_q^m(0)\right\} = \ln\left\{\pi_0^m\right\} = \ln\left\{1/M_k\right\}$$

Definition 2: The Metric from $\phi_{qi}(k)$ to $\phi_{qj}(k-1)$ is:

$$M\left\{\phi_{qj}(k-1) \to \phi_{qi}(k)\right\} = \ln\left\{\Pr ob\left(\phi_q(k) = \phi_{qi}(k)\right) | \Pr ob\left(\phi_q(k-1) = \phi_{qj}(k-1)\right)\right\}$$
$$+ \quad \ln\left\{p\left(z(k) | \phi_{qi}(k)\right)\right\}$$

Optimum Decision Rule for the Channel Phase Path (5/5)

Definition 3: The Metric of path H_m from $\phi_q^m(0)$ to $\phi_q^m(p)$ is:

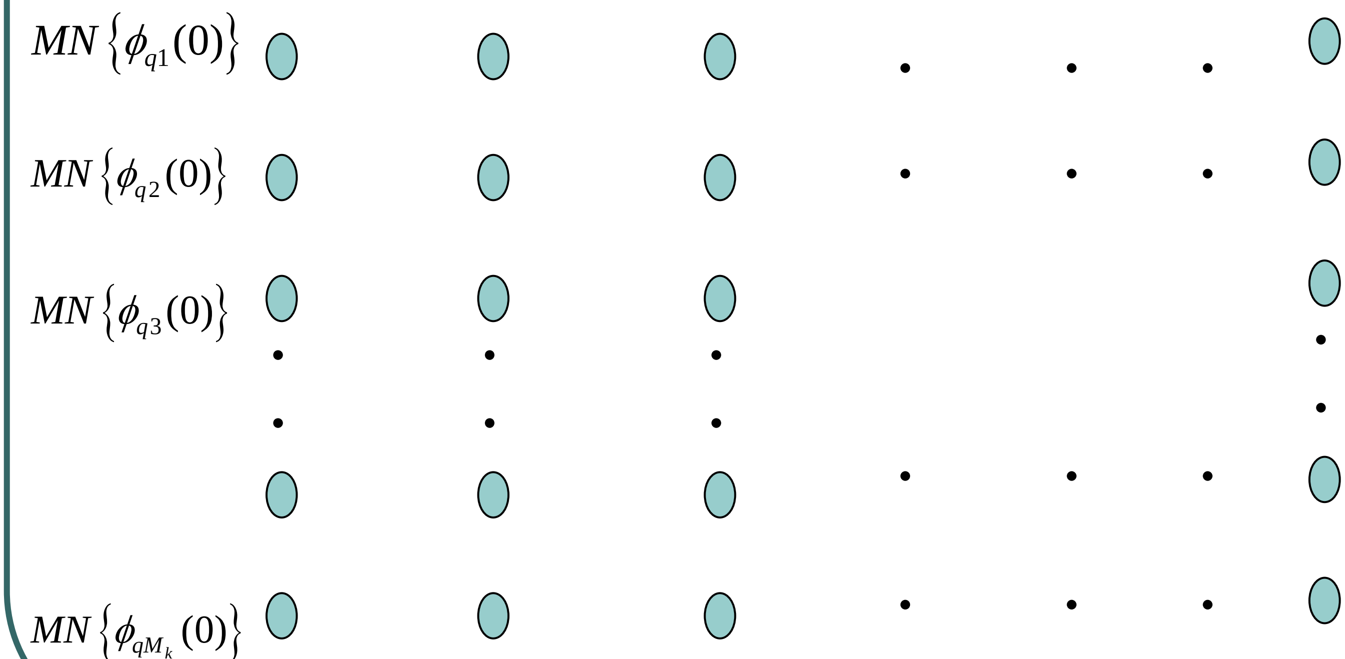
$$M\left(\phi_q^m(p)\right) = \ln\left\{\pi_0^m\right\} + \sum_{k=1}^{p}\left[\ln\left\{\pi_k^m\right\} + \ln\left\{p\left(z(k) \mid \phi_q^m(k)\right)\right\}\right]$$

→ We use Viterbi decoding Algorithm as the optimum decoding algorithm.

→ (VDA + FSM) is the optimum decoding based smoothing algorithm.

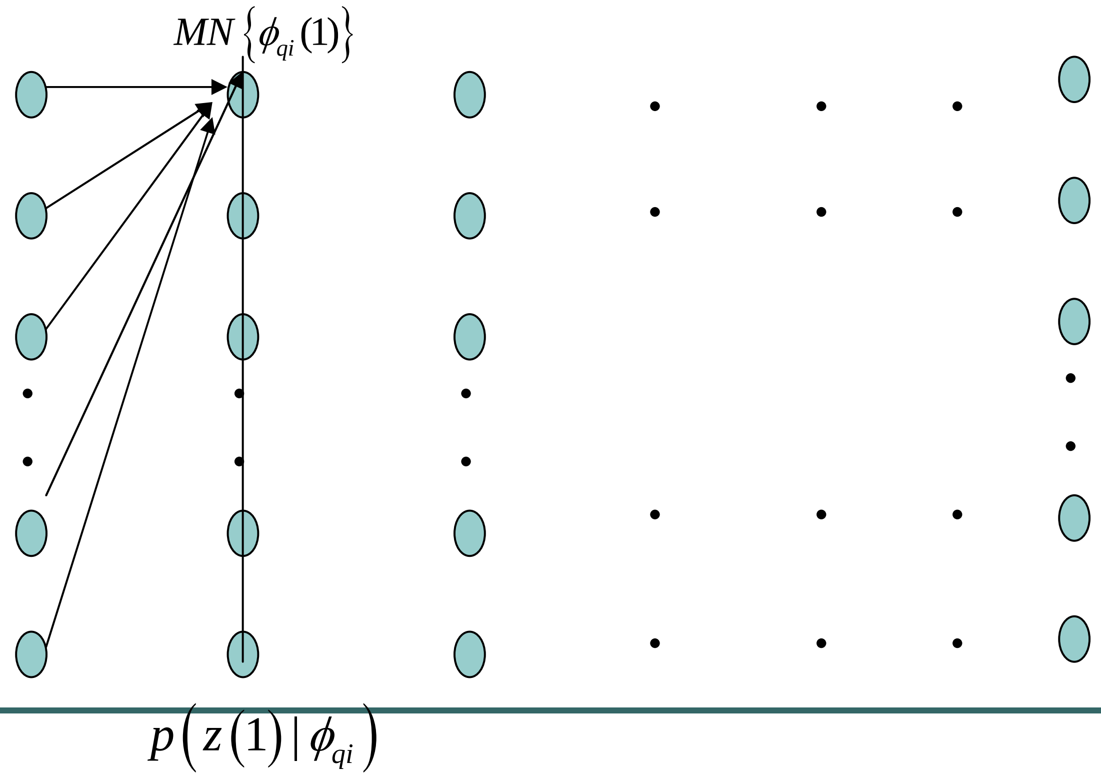
OPTIMUM DECODING BASED SMOOTHING ALGORITHM (1/3)

- First Step: Calculate $MN\left\{\phi_{qi}(0)\right\}$



OPTIMUM DECODING BASED SMOOTHING ALGORITHM (2/3)

- <u>Second Step:</u> Calculate $\pi_{ji}(1) = \ln\left\{\Pr ob\left(\phi_q(1) = \phi_{qi}(1) \mid \phi_q(0) = \phi_{qj}(0)\right)\right\}$ and $p(z(1) \mid \phi_{qi}(1))$ Select the largest Metric $MN\left\{\phi_{q1}(1)\right\}$ Discard the other paths

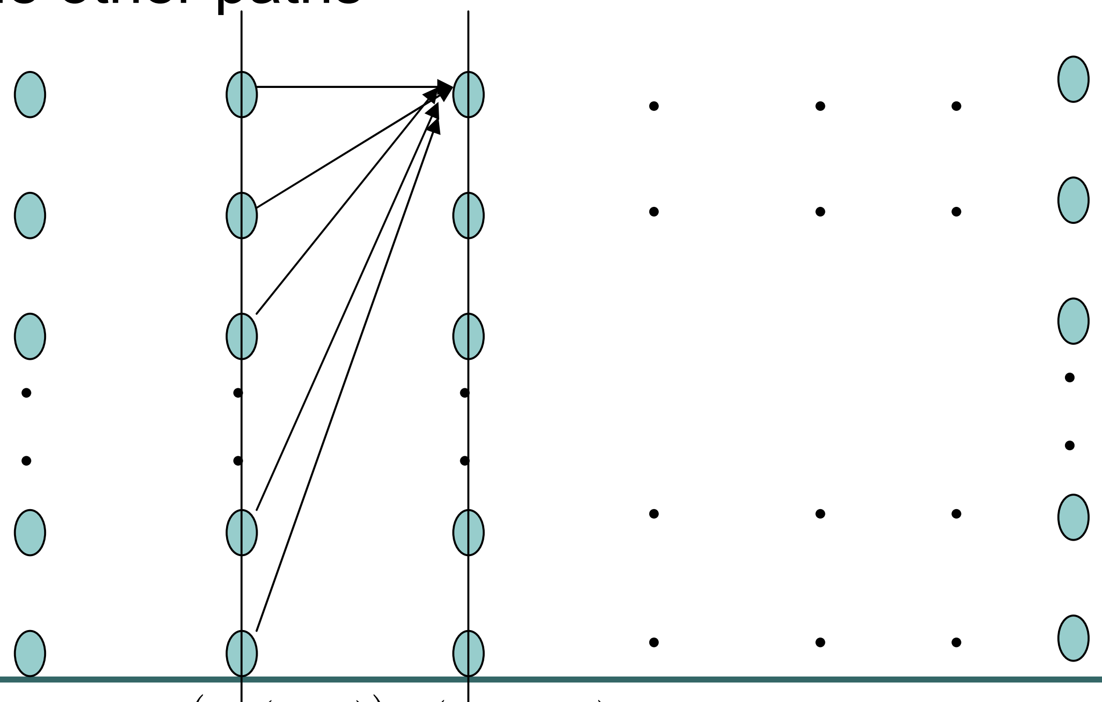


OPTIMUM DECODING BASED SMOOTHING ALGORITHM (3/3)

- <u>k'th Step:</u> Calculate $\pi_{ji}(k) = \ln\left\{\Pr ob\left(\phi_q(k) = \phi_{qi}(k) \mid \phi_q(k-1) = \phi_{qj}(k-1)\right)\right\}$

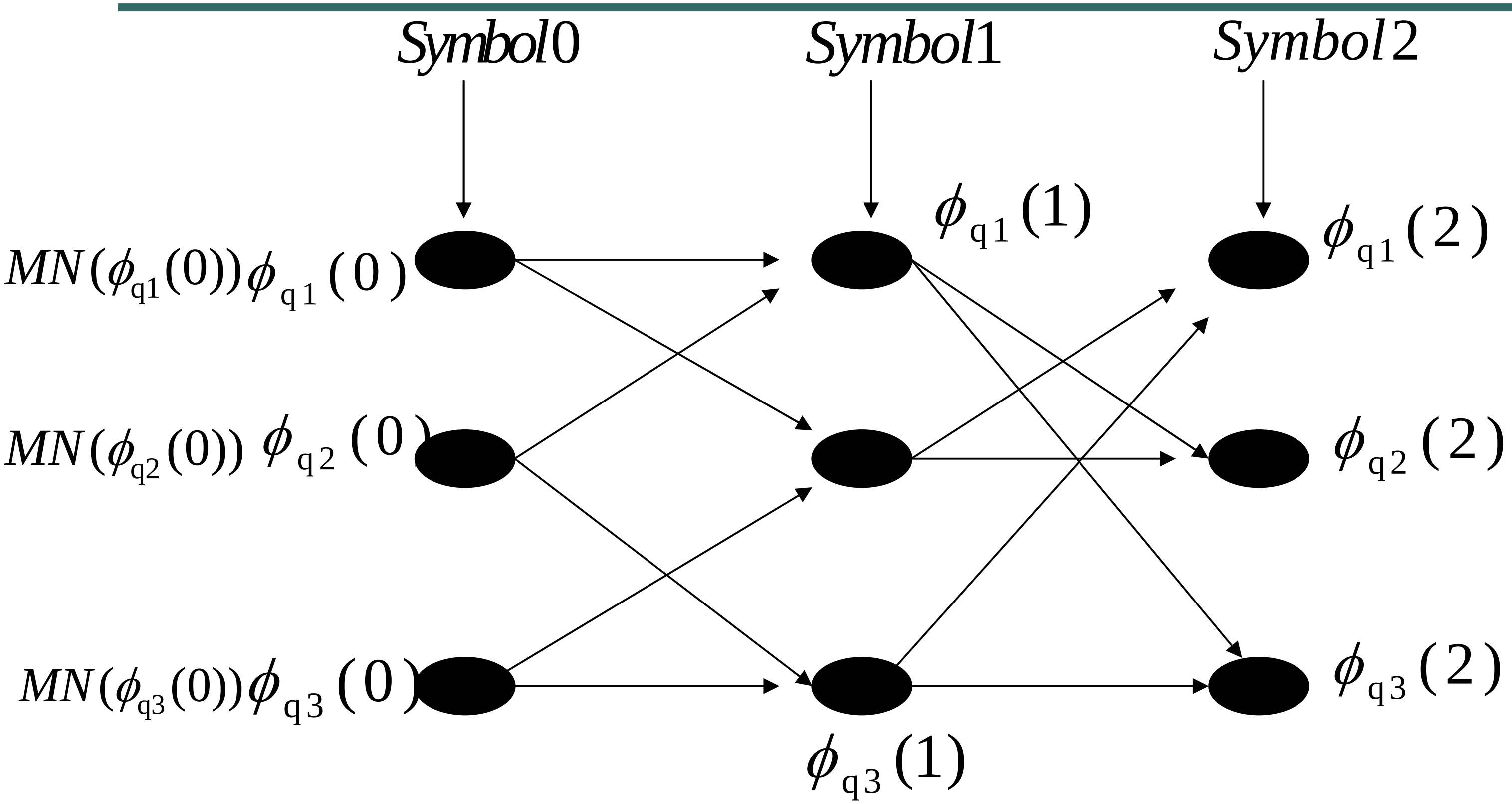
for k=2,....L Calculate $p(z(k) \mid \phi_{qi}(k))$ Select the largest Metric $MN\left\{\phi_{qi}(k)\right\}$

Discard the other paths



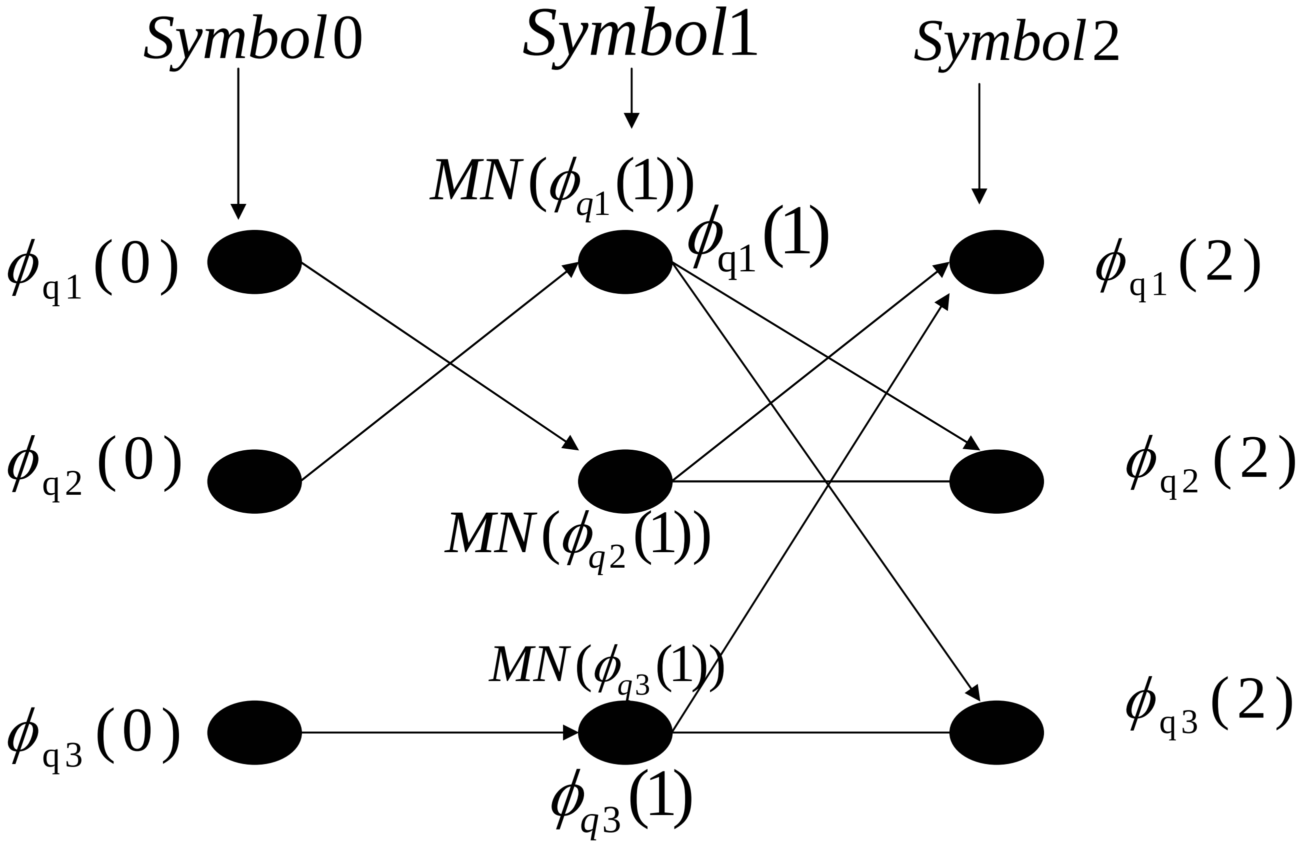
$MN\left(\phi_{qi}(k-1)\right)$ $p\left(z(k) \mid \phi_{qi}\right)$

An example of the ODSA (1/3)



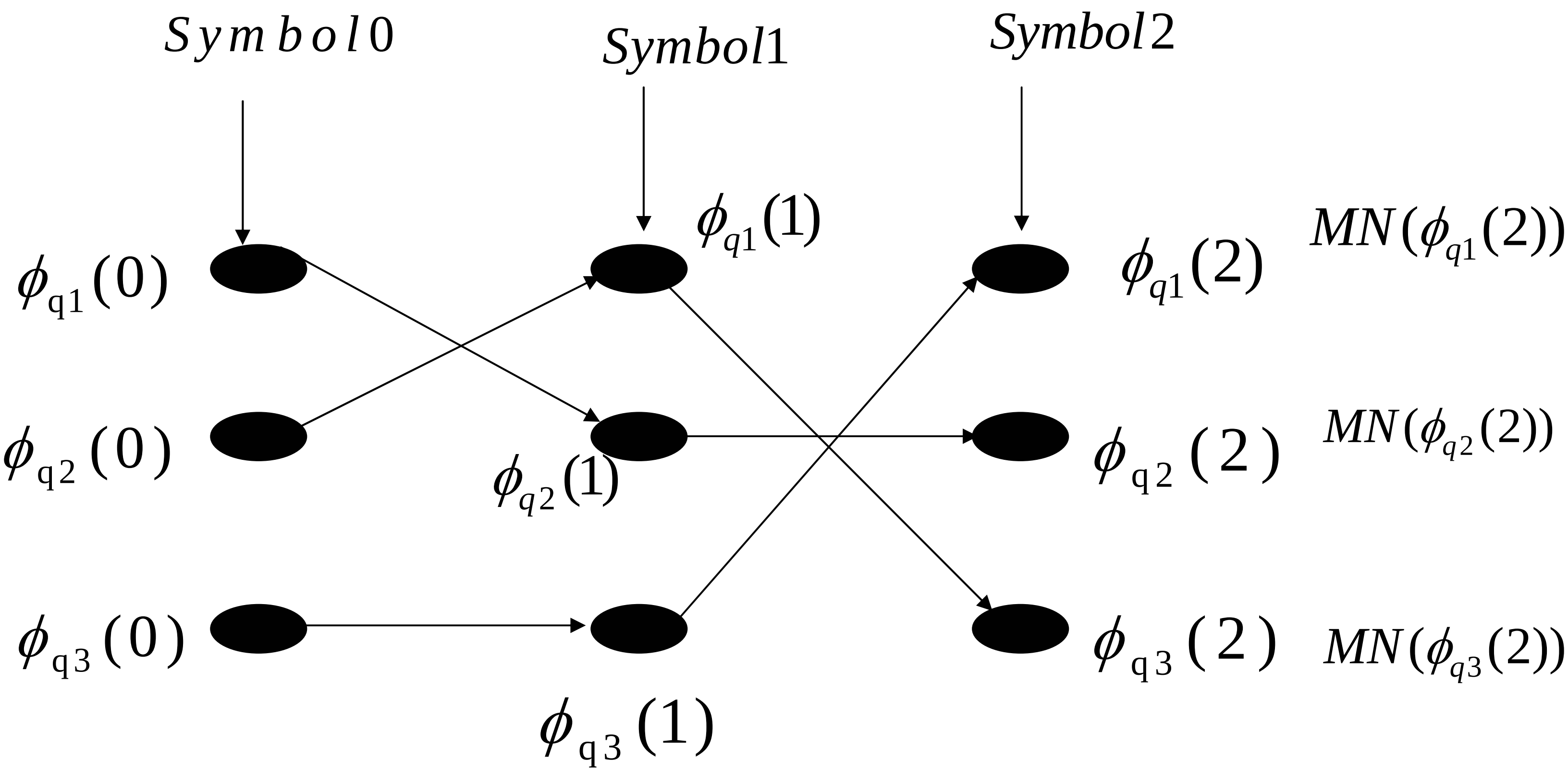
(a)

An example of the ODSA (2/3)



(b)

An example of the ODSA (3/3)



(c)

An Example with Interference and Gaussian disturbance and observation noises

Phase Transition Model : $\phi(k+1) = f\left(k, \phi(k), w(k)\right)$

Observation Model : $z\left(k\right) = g\left(k, \phi(k), \alpha\left(k\right)\right) + v\left(k\right)$

$f\left(k, \phi(k), w(k)\right)$	is vx1 vector
$g\left(k, \phi(k), \alpha(k)\right)$	is rx1 vector
$v\left(k\right)$	is bx1 vector with zero mean and covariance matrix $R_v(k)$
$\alpha(k)$	is mx1 interference vector with known statistics

The Metric of Branch beween the nodes $\phi_q(k-1)$ and $\phi_q(k)$ (1/2)

$$\mathrm{p}\left(\mathrm{z(k)}|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)=\mathrm{p}\left(\mathrm{z(k)}|\phi(\mathrm{k})=\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)$$

$$\mathrm{p}\left(\mathrm{z(k)}|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)=A\exp\left(-\frac{B}{2}\right)$$

$$A=\left(2\pi\right)^{-\frac{r}{2}}\det\left\{R_v(k)\right\}^{-\frac{1}{2}}$$

$$B=\left[z(k)-g\left(k,\phi(k),\alpha(k)\right)\right]^T\left[R_v(k)\right]^{-1}\left[z(k)-g\left(k,\phi(k),\alpha(k)\right)\right]$$

The Metric of Branch beween the nodes $\phi_q(k-1)$ and $\phi_q(k)$ (2/2)

$$\mathrm{p}\left(\mathrm{z(k)}|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k})\right)=\sum_{l=1}^{R_k}\mathrm{p}\left(\mathrm{z(k)}|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k}),\alpha_{dl}(\mathrm{k})\right)p(\alpha_{dl}(\mathrm{k}))$$

where $\mathrm{p}\left(\mathrm{z(k)}|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k}),\alpha_{dl}(\mathrm{k})\right)=\mathrm{p}\left(\mathrm{z(k)}|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})=\alpha_{dl}(\mathrm{k})\right)$

Therefore, the metric of the branch between $\phi_q^i(k-1)$ and $\phi_q^i(k)$ is:

$$M\left(\phi_q^i(k-1)\rightarrow\phi_q^i(k)\right)=\ln\left(\pi_k^i\right)+\ln\left(p\left(z(k)\,|\,\phi_q^i(k)\right)\right)$$

OUTLINE

1) Introduction
2) Multipath Fading Channel Characterization
3) Optimum Decoding Based Smoothing Algorithm (ODSA)
4) Optimum Receivers for BOFSK and BPSK in Slowly fading frequency-nonselective channels
5) Simulation Results
6) Conclusion & Future Work

OPTIMUM RECEIVER FOR BINARY SIGNALS IN SFFNC

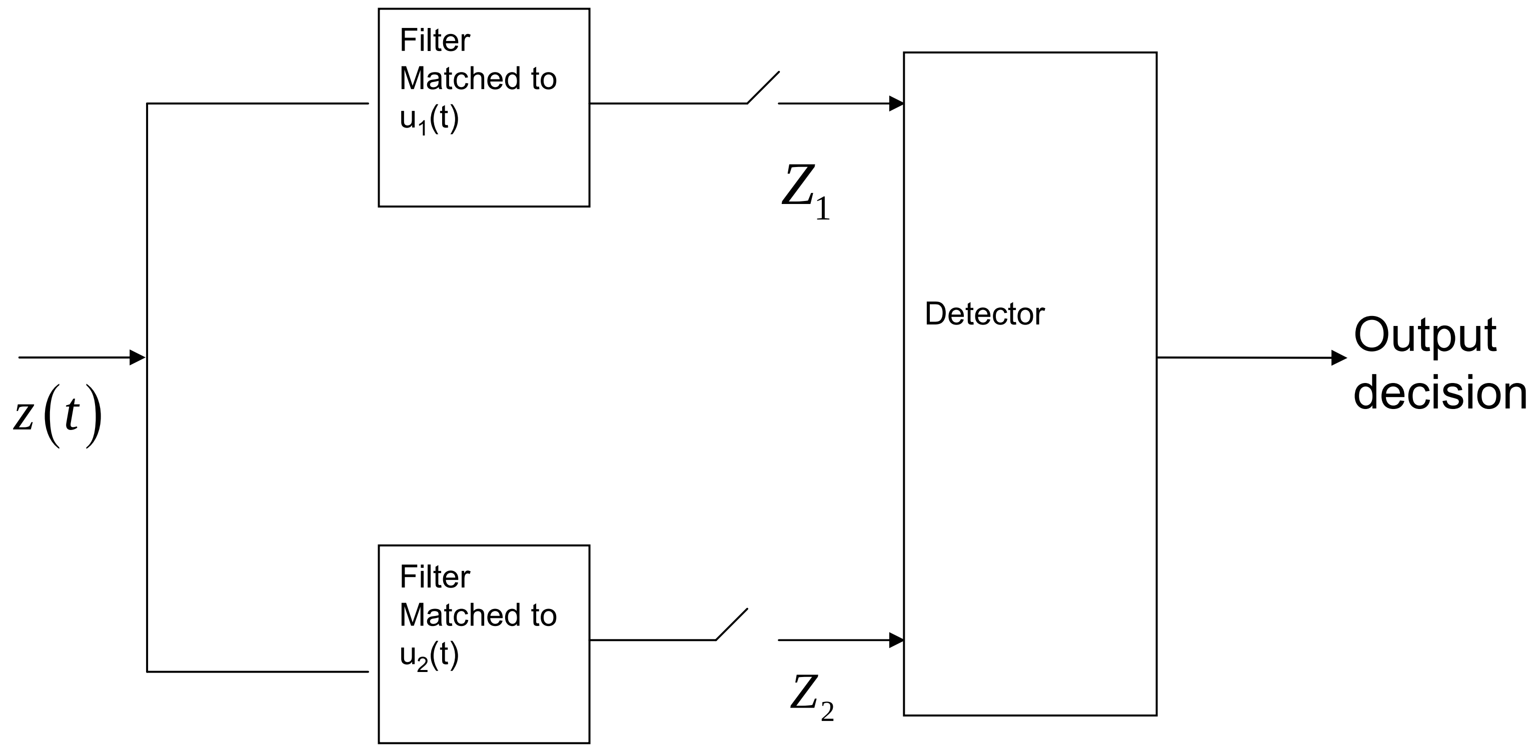
Binary bandpass signals: $s_m(t) = \mathrm{Re}\left\{u_m(t)\exp\left(j2\pi f_c t\right)\right\}$

$$\int_0^T \left|u_m(t)\right|^2 dt = 2E_m = 2E$$

Correlation coefficient: $$\rho = \frac{\int_0^T \mathrm{u}_1(t)\mathrm{u}_2^*(t)dt}{2E}$$

The received signal is: $z(t) = \alpha \exp(-j\phi)u_m(t) + v(t)$

OPTIMUM RECEIVER FOR BINARY SIGNALS (1/2)



$\vec{Z} = [Z_{1r} \quad Z_{1m} \quad Z_{2r} \quad Z_{2m}]$ where $Z_1 = Z_{1r} + jZ_{1m}$ and $Z_2 = Z_{2r} + jZ_{2m}$

OPTIMUM RECEIVER FOR BINARY SIGNALS (2/2)

If $u_1(t)$ is transmitted:

$$Z_1 = 2\alpha E\cos(\phi) + V_{1r} + j\left(-2\alpha E\sin(\phi) + V_{1m}\right)$$

$$Z_2 = \rho 2\alpha E\cos(\phi) + V_{2r} + j\left(-\rho 2\alpha E\sin(\phi) + V_{2m}\right)$$

If $u_2(t)$ is transmitted:

$$Z_1 = 2\alpha E\rho\cos(\phi) + V_{1r} + j\left(-2\alpha E\rho\sin(\phi) + V_{1m}\right)$$

$$Z_2 = 2\alpha E\cos(\phi) + V_{2r} + j\left(-2\alpha E\sin(\phi) + V_{2m}\right)$$

Optimum Decision Variables

$$p(u_m / \vec{Z}) = \frac{p(\vec{Z} / u_m) p(u_m)}{p(\vec{Z})} \qquad m = 1,2$$

The optimum decision rule is: $p(u_1 | \vec{Z}) \underset{u_2}{\overset{u_1}{\gtrless}} p(u_2 | \vec{Z})$

or $$\Omega(\vec{z}) = \frac{p(\vec{z} / u_1)}{p(\vec{z} / u_2)} \underset{u2}{\overset{u1}{\gtrless}} \frac{p(u_2)}{p(u_1)}$$

Optimum Coherent Detector for Binary Orthogonal Signals $\rho = 0$

Channel Phase is assumed to be known at the detector:

$$\vec{Z} = \begin{bmatrix} Z_{1r} & Z_{1m} & Z_{2r} & Z_{2m} \end{bmatrix}$$

$$\Omega\left(\vec{Z}\right) = \frac{p\left(\vec{Z} \mid u_1\right)}{p\left(\vec{Z} \mid u_2\right)} \underset{u_2}{\overset{u_1}{\gtrless}} \frac{p\left(u_1\right)}{p\left(u_2\right)} \quad \Rightarrow \quad p(\vec{Z} / u_m) = \int_{-\infty}^{\infty} p(\vec{Z} / u_m, \alpha) p(\alpha) d\alpha$$

u1(t) is transmitted $\Rightarrow$

$$Z_1 = Z_{1r} + jZ_{1m} = 2\alpha E \cos\left(\phi\right) + V_{1r} + j\left(-2\alpha E \sin\left(\phi\right) + V_{1m}\right)$$

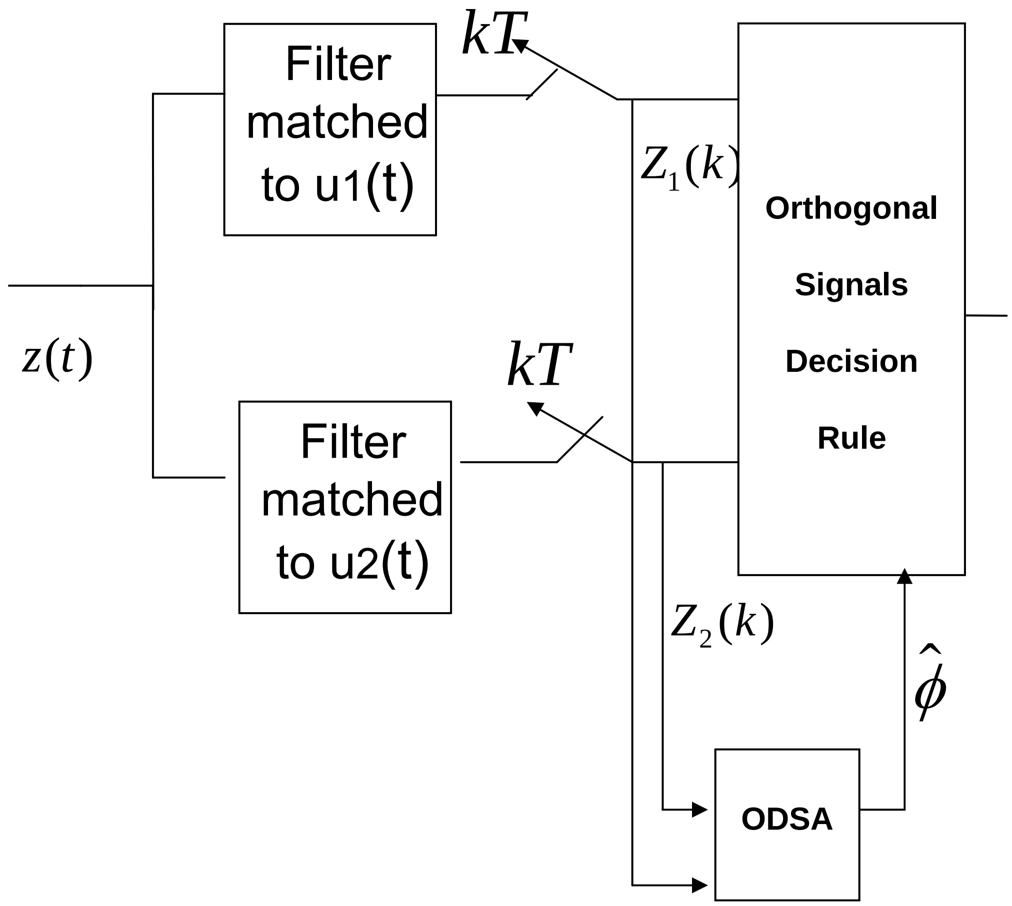
$$Z_2 = Z_{2r} + jZ_{2m} = V_{1r} + jV_{1m}$$

u2(t) is transmitted $\Rightarrow$

$$Z_1 = Z_{1r} + jZ_{1m} = V_{1r} + jV_{1m}$$

$$Z_2 = Z_{2r} + jZ_{2m} = 2\alpha E \cos\left(\phi\right) + V_{2r} + j\left(-2\alpha E \sin\left(\phi\right) + V_{2m}\right)$$

PROPOSED RECEIVER FOR BINARY ORTHOGONAL SIGNALS



Noise Variance

$H(f)$ is frequency response of the MF $u_m(T-t)$

$S_v(f)$ is PSD of MF output noise

$S_w(f) = N_0$ is PSD of white noise

Parseval's Theorem

$$\sigma^2 = \int_{-\infty}^{\infty} S_V(f)df = \int_{-\infty}^{\infty} Sw(f)\left|H(f)\right|^2 df$$

$$= N_0 \int_{-\infty}^{\infty} \left|H(f)\right|^2 df = N_0 \int_0^T \left|h(t)\right|^2 dt$$

$$= N_0 \int_0^T \left|u_m(T-t)\right|^2 dt = 2N_0 E$$

DECISION RULE FOR BINARY ORTHOGONAL SIGNALS

The optimum decision rule is:

$$\Omega(\vec{Z}) = \frac{\left(\int_0^\infty \alpha \exp\left[\alpha^2\left(-\frac{1}{2\sigma_1^2}-\frac{4E^2}{\sigma^2}\right)+\alpha\left(\frac{2E}{\sigma^2}\left(Z_{1r}\cos(\phi)-Z_{1m}\sin(\phi)\right)\right)\right]d\alpha\right)}{\left(\int_0^\infty \alpha \exp\left[\alpha^2\left(-\frac{1}{2\sigma_1^2}-\frac{4E^2}{\sigma^2}\right)+\alpha\left(\frac{2E}{\sigma^2}\left(Z_{2r}\cos(\phi)-Z_{2m}\sin(\phi)\right)\right)\right]d\alpha\right)} \underset{u_2}{\overset{u_1}{\gtrless}} \frac{p(u_2)}{p(u_1)}$$

If the apriori probabilities are equal:

$$\left[Z_{1r}\cos(\phi)-Z_{1m}\sin(\phi)\right] \underset{u_2}{\overset{u_1}{\gtrless}} \left[Z_{2r}\cos(\phi)-Z_{2m}\sin(\phi)\right] \quad \text{or} \quad \mathrm{Re}\left\{\exp(j\phi)\left(Z_1\right)\right\} \underset{u_2}{\overset{u_1}{\gtrless}} \mathrm{Re}\left\{\exp(j\phi)\left(Z_2\right)\right\}$$

Application of ODSA to Binary Orthogonal Signals (1/2)

Phase Transition Model:

$$\phi(k+1) = \tan^{-1}\left(\frac{Y(k+1)}{X(k+1)}\right) = \tan^{-1}\left(\frac{\lambda Y(k) + n_2(k)}{\lambda X(k) + n_1(k)}\right)$$

$$= \tan^{-1}\left(\frac{-\lambda\alpha(k)\sin\left(\phi\left(k\right)\right) + n_2(k)}{\lambda\alpha(k)\cos\left(\phi\left(k\right)\right) + n_1(k)}\right)$$

The output of the demodulators are:

$$Z_1(k) = \alpha(k)\exp(-j\phi(k))\left[2\beta E\right] + v_{1,1}(k) + jv_{1,2}(k)$$

$$Z_2(k) = \alpha(k)\exp(-j\phi(k))\left[(1-\beta)2E\right] + v_{2,1}(k) + jv_{2,2}(k)$$

$$\beta = \begin{cases} 1 & with \quad probability \quad 0.5 \quad if\ equally \quad likely \\ 0 & with \quad probability \quad 0.5 \quad if \quad equally \quad likely \end{cases}$$

Application of ODSA to Binary Orthogonal Signals (2/2)

In ODSA, we reduce these models:

Phase Transition Model:

$$\phi_q(k+1) = Q\left\{\tan^{-1}\left(\frac{Y(k+1)}{X(k+1)}\right)\right\} = Q\left\{\tan^{-1}\left(\frac{\lambda Y(k)+n_2(k)}{\lambda X(k)+n_1(k)}\right)\right\}$$

$$= Q\left\{\tan^{-1}\left(\frac{-\lambda\alpha(k)\sin\left(\phi_q(k)\right)+n_2(k)}{\lambda\alpha(k)\cos\left(\phi_q(k)\right)+n_1(k)}\right)\right\}$$

The Metric of Branch beween the nodes $\phi_q(k\text{-}1)$ and $\phi_q(k)$ in BOS(1/3)

Since $v_{1,1}(k)$, $v_{1,2}(k)$ and $v_{2,1}(k)$, $v_{2,2}(k)$ are uncorrelated Gaussian random variables with zero mean and variance σ^2

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k}),\mathrm{Z}_2(\mathrm{k})|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)=\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi(\mathrm{k})=\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)\mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi(\mathrm{k})=\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)$$

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k}),\mathrm{Z}_2(\mathrm{k})|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)=\left[\sum_{\beta=0,1}\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)\mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)p(\beta)\right]$$

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left|\mathrm{Z}_1(\mathrm{k})\text{-}2\alpha(\mathrm{k})\exp\left(\text{-j}\phi_q^i(\mathrm{k})\right)\beta E\right|^2}{2\sigma^2}\right)$$

$$\mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi_{\mathrm{q}}^{\mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left|\mathrm{Z}_2(\mathrm{k})\text{-}2\alpha(\mathrm{k})\exp\left(\text{-j}\phi_q^i(\mathrm{k})\right)(1-\beta)E\right|^2}{2\sigma^2}\right)$$

The Metric of Branch beween the nodes $\phi_q(k-1)$ and $\phi_q(k)$ in BOS(2/3)

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k})\right) = \sum_{\beta=0,1}\sum_{l=1}^{R_k}\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k}),\beta,\alpha_{dl}(k)\right)p\left(\alpha_{dl}(k)\right)p\left(\beta\right)$$

$$\mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k})\right) = \sum_{\beta=0,1}\sum_{l=1}^{R_k}\mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k}),\beta,\alpha_{dl}(k)\right)p\left(\alpha_{dl}(k)\right)p\left(\beta\right)$$

where $\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k}),\beta,\alpha_{dl}(k)\right) = \mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k}),\beta,\alpha(k)=\alpha_{dl}(k)\right)$

and $\mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k}),\beta,\alpha_{dl}(k)\right) = \mathrm{p}\left(\mathrm{Z}_2(\mathrm{k})|\phi_\mathrm{q}{}^\mathrm{i}(\mathrm{k}),\beta,\alpha(k)=\alpha_{dl}(k)\right)$

The Metric of Branch beween the nodes $\phi_q(k-1)$ and $\phi_q(k)$ in BOS(3/3)

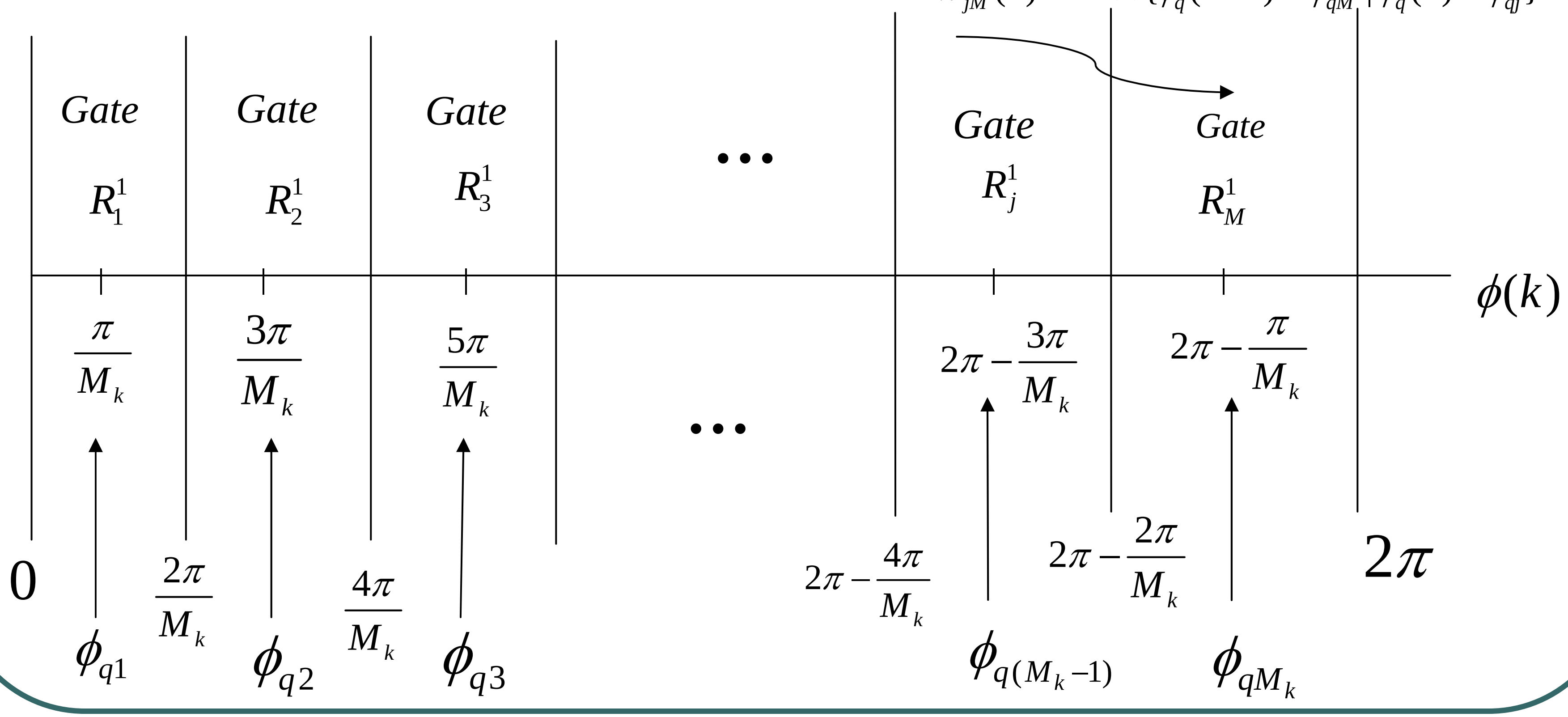
Therefore, the Metric of a Branch between the nodes $\phi_q(k-1)$ and $\phi_q(k)$ is:

$$\Rightarrow \quad M(\phi_q^i(k-1) \rightarrow \phi_q^i(k)) = \ln(\pi_k^i) + \ln(p(Z_1(k), Z_2(k) \mid \phi_q^i(k)))$$

QUANTIZATION OF CHANNEL PHASE

$$\phi_q(k+1) = Q\left\{ f\left(k, \phi_q(k), w_d(k)\right)\right\}$$

$$\pi_{jM}(k) = \Pr ob\{\phi_q(k+1) = \phi_{qM} \mid \phi_q(k) = \phi_{qj}\}$$



Transition Probabilities from simulation of first order Markov fading Process in MATLAB (1/4)

$$\phi(k) = \tan^{-1}\left(\frac{Y(k)}{X(k)}\right) \qquad for \qquad k = 1, \ldots\ldots\ldots\ldots, L$$

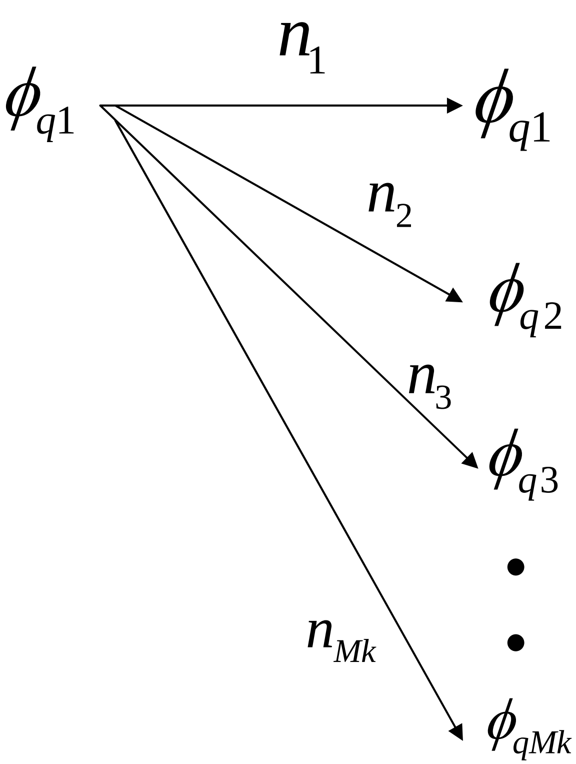
$$\phi_q(k) = Q\{\phi(k)\} \qquad for \qquad k = 1, \ldots\ldots\ldots\ldots, L$$

$$\phi_q = \{\phi_{q1}, \phi_{q1}, \phi_{q4}, \phi_{q2}, \phi_{q2}, \phi_{q1}, \phi_{q3}, \phi_{qMk}, \phi_{q2}, \ldots\ldots\ldots\ldots\ldots\ldots, \phi_{q3}\}$$

$k = 1$ $k = 2$ $k = 3$ $k = L$

Transition Probabilities from simulation of first order Markov fading Process in MATLAB (2/4)

$L = 1000 \quad bits \quad with \quad 2000 \quad runs$



$$\Pr ob\{\phi_{q1} \to \phi_{q1}\} = \frac{n_1}{n_1 + n_2 + n_3 + \ldots\ldots + n_{Mk}}$$

$$\Pr ob\{\phi_{q1} \to \phi_{q2}\} = \frac{n_2}{n_1 + n_2 + n_3 + \ldots\ldots + n_{Mk}}$$

• • •

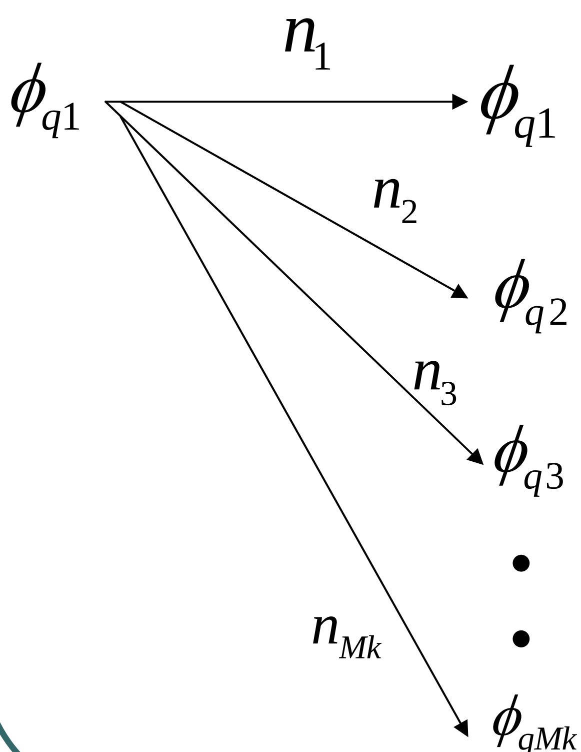
$$\Pr ob\{\phi_{q1} \to \phi_{qMk}\} = \frac{n_{Mk}}{n_1 + n_2 + n_3 + \ldots\ldots + n_{Mk}}$$

Transition Probabilities from simulation of first order Markov fading Process in MATLAB (3/4)

$\lambda = 1$

$$X(k+1) = X(k)$$
$$Y(k+1) = Y(k)$$

$$k = 1, \ldots\ldots\ldots, L$$



$$\Pr ob\{\phi_{q1} \rightarrow \phi_{q1}\} = \frac{n_1}{n_1 + n_2 + n_3 + \ldots\ldots + n_{Mk}} = 1$$

$$\Pr ob\{\phi_{q1} \rightarrow \phi_{q2}\} = \frac{n_2}{n_1 + n_2 + n_3 + \ldots\ldots + n_{Mk}} = 0$$

• • •

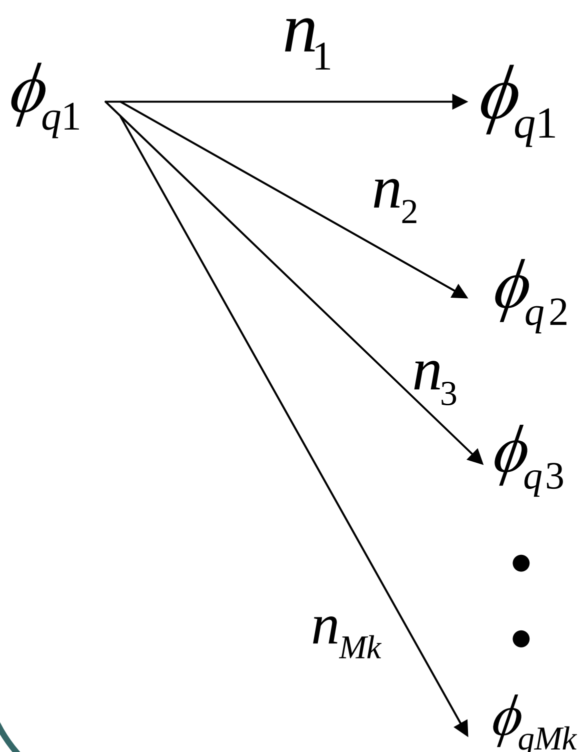
$$\Pr ob\{\phi_{q1} \rightarrow \phi_{qMk}\} = \frac{n_{Mk}}{n_1 + n_2 + n_3 + \ldots\ldots + n_{Mk}} = 0$$

Transition Probabilities from simulation of first order Markov fading Process in MATLAB (4/4)

$\lambda = 0$ ⇒

$$X(k+1) = n_1(k)$$
$$Y(k+1) = n_2(k)$$

$\leftarrow N(0,\gamma)$



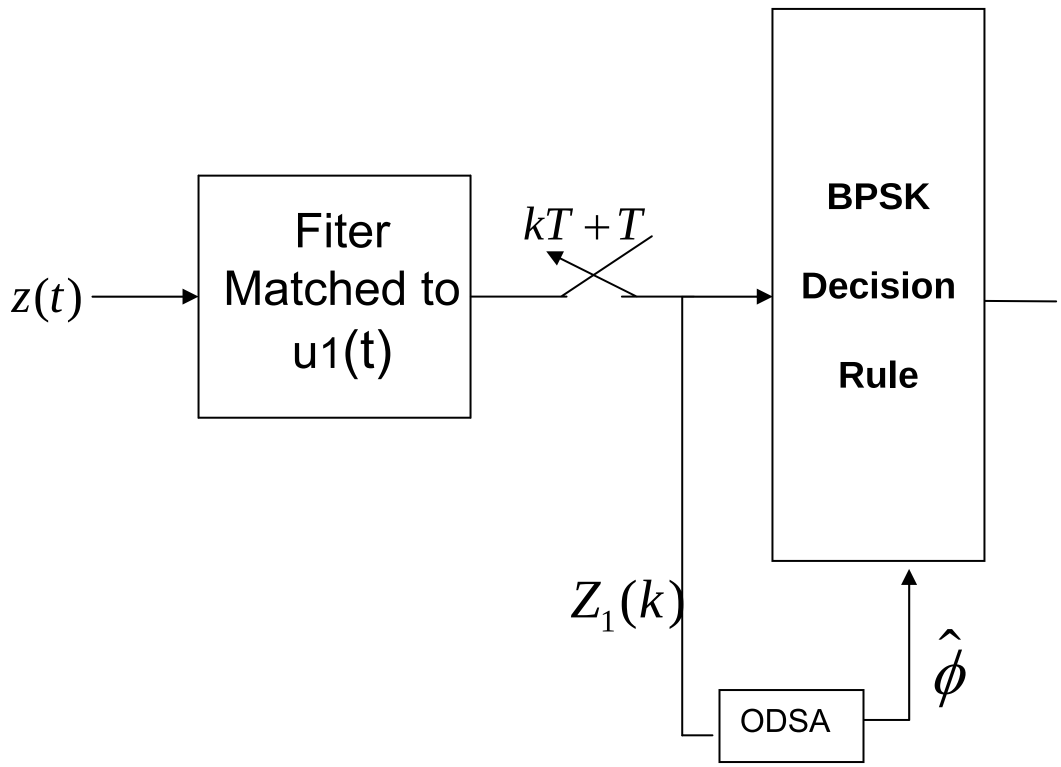
$$\Pr ob\{\phi_{q1} \to \phi_{q1}\} = \frac{n_1}{n_1 + n_2 + n_3 + \ldots.. + n_{Mk}} = \frac{1}{M_k}$$

$$\Pr ob\{\phi_{q1} \to \phi_{q2}\} = \frac{n_2}{n_1 + n_2 + n_3 + \ldots.. + n_{Mk}} = \frac{1}{M_k}$$

• • •

$$\Pr ob\{\phi_{q1} \to \phi_{qMk}\} = \frac{n_{Mk}}{n_1 + n_2 + n_3 + \ldots.. + n_{Mk}} = \frac{1}{M_k}$$

PROPOSED RECEIVER FOR BPSK



Optimum Coherent Detector for BPSK Signals $\rho = -1$

Channel Phase is assumed to be known at the detector:

$$\vec{Z} = [Z_{1r} \quad Z_{1m}]$$

$$\Omega(\vec{Z}) = \frac{p(\vec{Z} \mid u_1)}{p(\vec{Z} \mid u_2)} \underset{u_2}{\overset{u_1}{\gtrless}} \frac{p(u_1)}{p(u_2)} \quad \Rightarrow \quad p(\vec{Z}/u_m) = \int_{-\infty}^{\infty} p(\vec{Z}/u_m, \alpha) p(\alpha) d\alpha$$

u1(t) is transmitted $\Rightarrow$

$$Z_1 = Z_{1r} + jZ_{1m}$$
$$= 2\alpha E \cos(\phi) + V_{1r} + j(-2\alpha E \sin(\phi) + V_{1m})$$

u2(t) is transmitted $\Rightarrow$

$$Z_1 = Z_{1r} + jZ_{1m}$$
$$= -2\alpha E \cos(\phi) + V_{2r} + j(2\alpha E \sin(\phi) + V_{2m})$$

DECISION RULE FOR BPSK

The optimum decision rule is:

$$\Omega(\vec{Z}) = \frac{\left(\int_0^\infty \alpha \exp\left[\alpha^2\left(-\frac{1}{2\sigma_1^2}-\frac{4E^2}{\sigma^2}\right)+\alpha\left(\frac{2E}{\sigma^2}\left(Z_{1r}\cos(\phi)-Z_{1m}\sin(\phi)\right)\right)\right]d\alpha\right)}{\left(\int_0^\infty \alpha \exp\left[\alpha^2\left(-\frac{1}{2\sigma_1^2}-\frac{4E^2}{\sigma^2}\right)+\alpha\left(-\frac{2E}{\sigma^2}\left(Z_{1r}\cos(\phi)-Z_{1m}\sin(\phi)\right)\right)\right]d\alpha\right)} \underset{u_2}{\overset{u_1}{\gtrless}} \frac{p(u_2)}{p(u_1)}$$

If the apriori probabilities are equal:

$$\left[Z_{1r}\cos(\phi)-Z_{1m}\sin(\phi)\right] \underset{u_2}{\overset{u_1}{\gtrless}} 0$$

or

$$\mathrm{Re}\left\{\exp(j\phi)\left(Z_1\right)\right\} \underset{u_2}{\overset{u_1}{\gtrless}} 0$$

Application of ODSA to BPSK Signals (1/2)

Phase Transition Model:

$$\phi(k+1) = \tan^{-1}\left(\frac{Y(k+1)}{X(k+1)}\right) = \tan^{-1}\left(\frac{\lambda Y(k) + n_2(k)}{\lambda X(k) + n_1(k)}\right)$$

$$= \tan^{-1}\left(\frac{-\lambda \alpha(k)\sin\left(\phi(k)\right) + n_2(k)}{\lambda \alpha(k)\cos\left(\phi(k)\right) + n_1(k)}\right)$$

Observation Model:

$$z(t) = \alpha(t)\exp(-j\phi(t))\left[\beta u_1(t) + \left(1-\beta\right)u_2(t)\right] + v(t)$$

After Matched Filtering and sampling:

$$Z_1(k) = \alpha(k)\exp(-j\phi(k))\left[\left(2\beta - 1\right)2E\right] + v_{1,1}(k) + jv_{1,2}(k)$$

$$\beta = \begin{cases} 1 \quad with \quad probability \quad 0.5 \quad if\ equally \quad likely \\ 0 \quad with \quad probability \quad 0.5 \quad if \quad equally \quad likely \end{cases}$$

Application of ODSA to BPSK Signals (2/2)

In ODSA, we reduce these models:

Phase Transition Model:

$$\phi_q\left(k+1\right)=Q\left\{\tan^{-1}\left(\frac{Y(k+1)}{X(k+1)}\right)\right\}=Q\left\{\tan^{-1}\left(\frac{\lambda Y(k)+n_2(k)}{\lambda X(k)+n_1(k)}\right)\right\}$$

$$=Q\left\{\tan^{-1}\left(\frac{-\lambda\alpha(k)\sin\left(\phi_q\left(k\right)\right)+n_2(k)}{\lambda\alpha(k)\cos\left(\phi_q\left(k\right)\right)+n_1(k)}\right)\right\}$$

The Metric of Branch beween the nodes $\phi_q(k\text{-}1)$ and $\phi_q(k)$ in BPSK(1/3)

where $v_{1,1}(k)$ and $v_{1,2}(k)$ are uncorrelated Gaussian random variables with zero mean and variance σ^2

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}^{\ \mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)=\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi(\mathrm{k})=\phi_\mathrm{q}^{\ \mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)$$

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}^{\ \mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k})\right)=\left[\sum_{\beta=0,1}\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}^{\ \mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)p\left(\beta\right)\right]$$

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_\mathrm{q}^{\ \mathrm{i}}(\mathrm{k}),\alpha(\mathrm{k}),\beta\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left|\mathrm{Z}_1(\mathrm{k})\text{-}\alpha(\mathrm{k})\exp\left(\text{-j}\phi_q^i(\mathrm{k})\right)\left(2\beta-1\right)2E\right|}{2\sigma^2}\right)$$

The Metric of Branch beween the nodes $\phi_q(k-1)$ and $\phi_q(k)$ in BPSK(2/3)

After approximating $\alpha(k)$ by a discrete random vector $\alpha_d(k)$;

$$\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k})\right) = \sum_{\beta=0,1}\sum_{l=1}^{R_k}\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k}),\beta,\alpha_{dl}(k)\right)p\left(\alpha_{dl}(k)\right)p\left(\beta\right)$$

where $\mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k}),\beta,\alpha_{dl}(k)\right) = \mathrm{p}\left(\mathrm{Z}_1(\mathrm{k})|\phi_{\mathrm{q}}{}^{\mathrm{i}}(\mathrm{k}),\beta,\alpha(k)=\alpha_{dl}(k)\right)$

The Metric of Branch beween the nodes $\phi_q(k-1)$ and $\phi_q(k)$ in BPSK(3/3)

Therefore, the Metric of a Branch between the nodes $\phi_q(k-1)$ and $\phi_q(k)$ is:

$$M(\phi_q^i(k-1) \to \phi_q^i(k)) = \ln(\pi_k^i) + \ln(p(Z_1(k) \mid \phi_q^i(k)))$$

Optimum NonCoherent Detector for Binary Orthogonal Signals

Channel Phase is assumed to be unknown at the detector:

$$\vec{Z} = \left[Z_{1r} \quad Z_{1m} \quad Z_{2r} \quad Z_{2m} \right]$$

$$p(\vec{Z}/u_m) = \int_0^{2\pi} \int_{-\infty}^{\infty} p(\vec{Z}/u_m, \phi, \alpha) p(\alpha) p(\phi) d\alpha d\phi \quad \Rightarrow \quad \Omega\left(\vec{Z}\right) = \frac{p\left(\vec{Z} \mid u_1\right)}{p\left(\vec{Z} \mid u_2\right)} \underset{u_2}{\overset{u_1}{\gtrless}} \frac{p\left(u_1\right)}{p\left(u_2\right)}$$

u1(t) is transmitted ⇒

$$Z_1 = Z_{1r} + jZ_{1m} = 2\alpha E \cos\left(\phi\right) + V_{1r} + j\left(-2\alpha E \sin\left(\phi\right) + V_{1m}\right)$$

$$Z_2 = Z_{2r} + jZ_{2m} = V_{1r} + jV_{1m}$$

u2(t) is transmitted ⇒

$$Z_1 = Z_{1r} + jZ_{1m} = V_{1r} + jV_{1m}$$

$$Z_2 = Z_{2r} + jZ_{2m} = 2\alpha E \cos\left(\phi\right) + V_{2r} + j\left(-2\alpha E \sin\left(\phi\right) + V_{2m}\right)$$

Noncoherent Decision Rules for Orthogonal Signals (1/2)

$$\Omega(\vec{Z}) = \frac{\left(\int_0^\infty \alpha \exp\left[\alpha^2\left(-\frac{1}{2\sigma_1^2}-\frac{4E^2}{\sigma^2}\right)\right] I_0\left(\frac{2E\alpha}{\sigma^2}\sqrt{(Z_{1r})^2+(Z_{1m})^2}\right)d\alpha\right)}{\left(\int_0^\infty \alpha \exp\left[\alpha^2\left(-\frac{1}{2\sigma_1^2}-\frac{4E^2}{\sigma^2}\right)\right] I_0\left(\frac{2E\alpha}{\sigma^2}\sqrt{(Z_{2r})^2+(Z_{2m})^2}\right)d\alpha\right)} \underset{u_2}{\overset{u_1}{\gtrless}} \frac{p(u_2)}{p(u_1)}$$

where $I_0(x)$ is defined as:

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp\left(x\cos(\theta)\right)d\theta$$

Noncoherent Decision Rules for Orthogonal Signals (2/2)

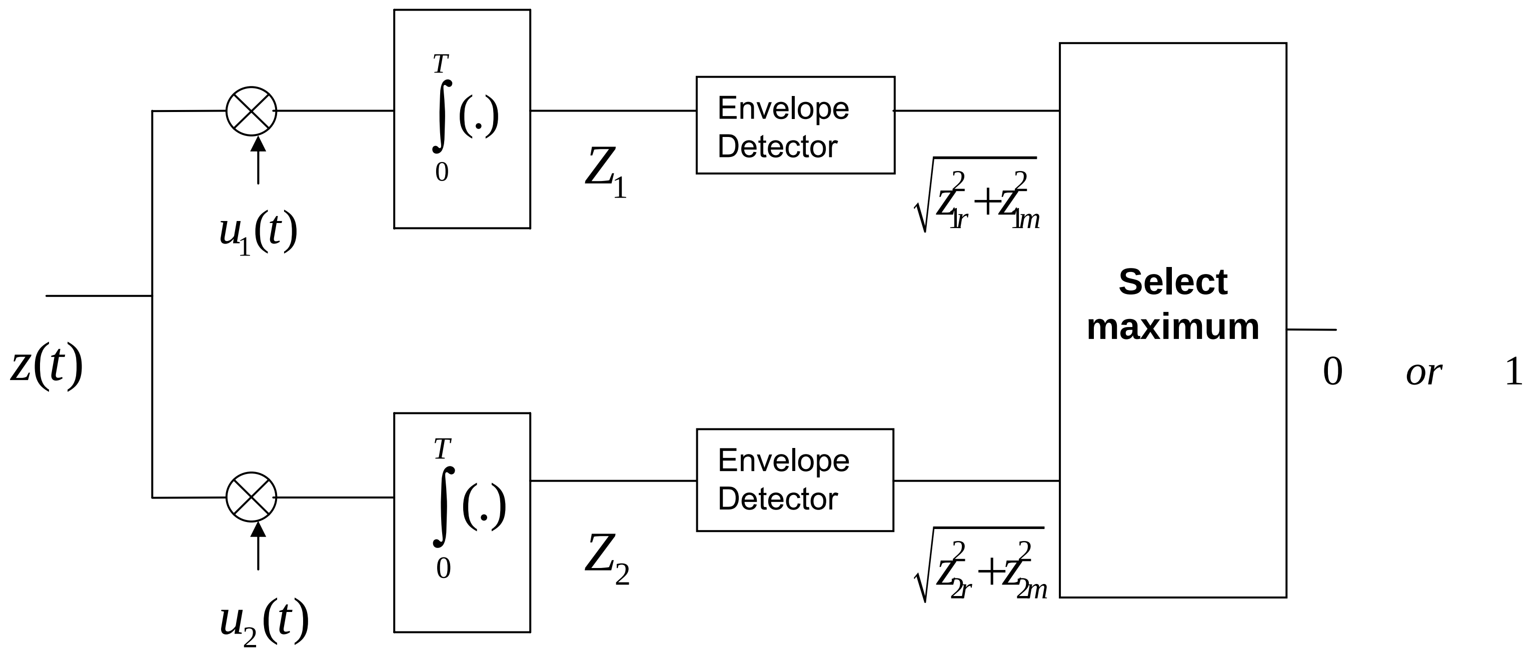
If signals are equally likely;

$$\sqrt{(Z_{1r})^2 + (Z_{1m})^2} \underset{u_2}{\overset{u_1}{\gtrless}} \sqrt{(Z_{2r})^2 + (Z_{2m})^2}$$

or

$$\left| \int_0^T z(t) u_1^*(t) dt \right| \underset{u_2}{\overset{u_1}{\gtrless}} \left| \int_0^T z(t) u_2^*(t) dt \right|$$

OPTIMUM NONCOHERENT BOS & BOFSK RECEIVER



Phase Estimation with Gaussian Random Walk Phase Transition model (1/7)

Phase Transition Model: $$\phi(k+1) = \phi(k) + w(k)$$

Observation Model $$z(t) = A\exp(-j\phi(t))\left[\beta u_1(t) + (1-\beta)u_2(t)\right] + v(t)$$

$$\beta = \begin{cases} 1 \quad with \quad probability \quad 0.5 \quad if\ equally \quad likely \\ 0 \quad with \quad probability \quad 0.5 \quad if \quad equally \quad likely \end{cases}$$

Phase Estimation with Gaussian Random Walk Phase Transition model (2/7)

For Binary Orthogonal Signals:

Phase Transition Model: $\phi_q\left(k+1\right)=Q\left\{\phi_q\left(k\right)+w_d\left(k\right)\right\}$

Matched Filter Output:

$$Z_1(k)=A\exp(-j\phi(k))\left[\beta 2E\right]+v_{1,1}(k)+jv_{1,2}(k)$$

$$Z_2(k)=A\exp(-j\phi(k))\left[(1-\beta)2E\right]+v_{2,1}(k)+jv_{2,2}(k)$$

Phase Estimation with Gaussian Random Walk Phase Transition model (3/7)

$$p\left(Z_1(k),Z_2(k)|\phi_q^{\,i}(k),\beta\right) =$$
$$p\left(Z_1(k)|\phi(k)=\phi_q^{\,i}(k),\beta\right)p\left(Z_2(k)|\phi(k)=\phi_q^{\,i}(k),\beta\right)$$

$$p\left(Z_1(k),Z_2(k)|\phi_q^{\,i}(k)\right) =$$
$$\left[\sum_{\beta=0,1} p\left(Z_1(k)|\phi_q^{\,i}(k),\beta\right)p\left(Z_2(k)|\phi_q^{\,i}(k),\beta\right)p\left(\beta\right)\right]$$

$$p\left(Z_1(k)|\phi_q^{\,i}(k),\beta\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left|Z_1(k)\text{-}A\exp\left(\text{-}j\phi_q^i(k)\right)\beta 2E\right|^2}{2\sigma^2}\right)$$

$$p\left(Z_2(k)|\phi_q^{\,i}(k),\beta\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left|Z_2(k)\text{-}A\exp\left(\text{-}j\phi_q^i(k)\right)(1-\beta)2E\right|^2}{2\sigma^2}\right)$$

Phase Estimation with Gaussian Random Walk Phase Transition model (4/7)

Therefore, the Metric of a Branch between the nodes $\phi_q(k-1)$ and $\phi_q(k)$ is:

$$M(\phi_q^i(k-1) \rightarrow \phi_q^i(k)) = \ln(\pi_k^i) + \ln(p(Z_1(k), Z_2(k) \mid \phi_q^i(k)))$$

Phase Estimation with Gaussian Random Walk Phase Transition model (5/7)

For BPSK Signals:

Phase Transition Model: $\phi_q\left(k+1\right)=Q\left\{\phi_q\left(k\right)+w_d\left(k\right)\right\}$

Matched Filter Output:

$$Z_1(k)=A\exp(-j\phi(k))\left[\left(2\beta-1\right)2E\right]+v_{1,1}(k)+jv_{1,2}(k)$$

Phase Estimation with Gaussian Random Walk Phase Transition model (6/7)

$$\Rightarrow \quad p\left(Z_1(k)|\phi_q^{\,i}(k)\right)=p\left(Z_1(k)|\phi(k)=\phi_q^{\,i}(k)\right)$$

$$\Rightarrow \quad p\left(Z_1(k)|\phi_q^{\,i}(k)\right)=\left[\sum_{\beta=0,1} p\left(Z_1(k)|\phi_q^{\,i}(k),\beta\right)p\left(\beta\right)\right]$$

$$\Rightarrow \quad p\left(Z_1(k)|\phi_q^{\,i}(k),\beta\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left|Z_1(k)-A\exp\left(-j\phi_q^i(k)\right)(2\beta-1)2E\right|^2}{2\sigma^2}\right)$$

Phase Estimation with Gaussian Random Walk Phase Transition model(7/7)

Therefore, the Metric of a Branch between the nodes $\phi_q(k-1)$ and $\phi_q(k)$ is:

$$M(\phi_q^i(k-1) \rightarrow \phi_q^i(k)) = \ln(\pi_k^i) + \ln(p(Z_1(k) \mid \phi_q^i(k)))$$

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5) **Simulation Results**
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SIMULATION RESULTS

- Binary Frequency Shift Keying

$$BOFSK \quad signals \quad s_m(t) = \cos 2\pi(f_c \pm \Delta f)t, \quad 0 \le t \le T$$

$\Delta f = \frac{1}{T}$ is min. frequency seperation for orthogonality of noncoherent FSK signals

$\Delta f = \frac{1}{2T}$ is min. seperation for orthogonality of coherent FSK signals

SIMULATION RESULTS

BOFSK Signals

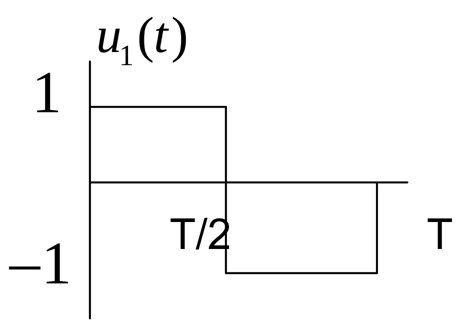
$$s_1(t) = \cos\left(2\pi(f_c + \frac{1}{2T})t\right)$$

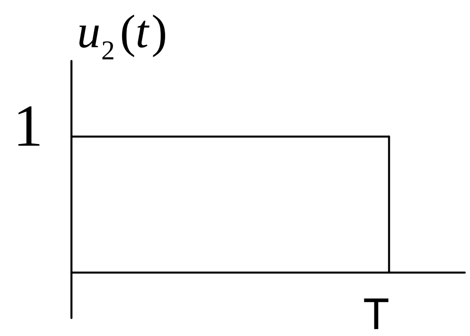
$$s_2(t) = \cos\left(2\pi(f_c - \frac{1}{2T})t\right)$$

$$u_1(t) = \exp\left(-j\pi\frac{t}{T}\right)$$

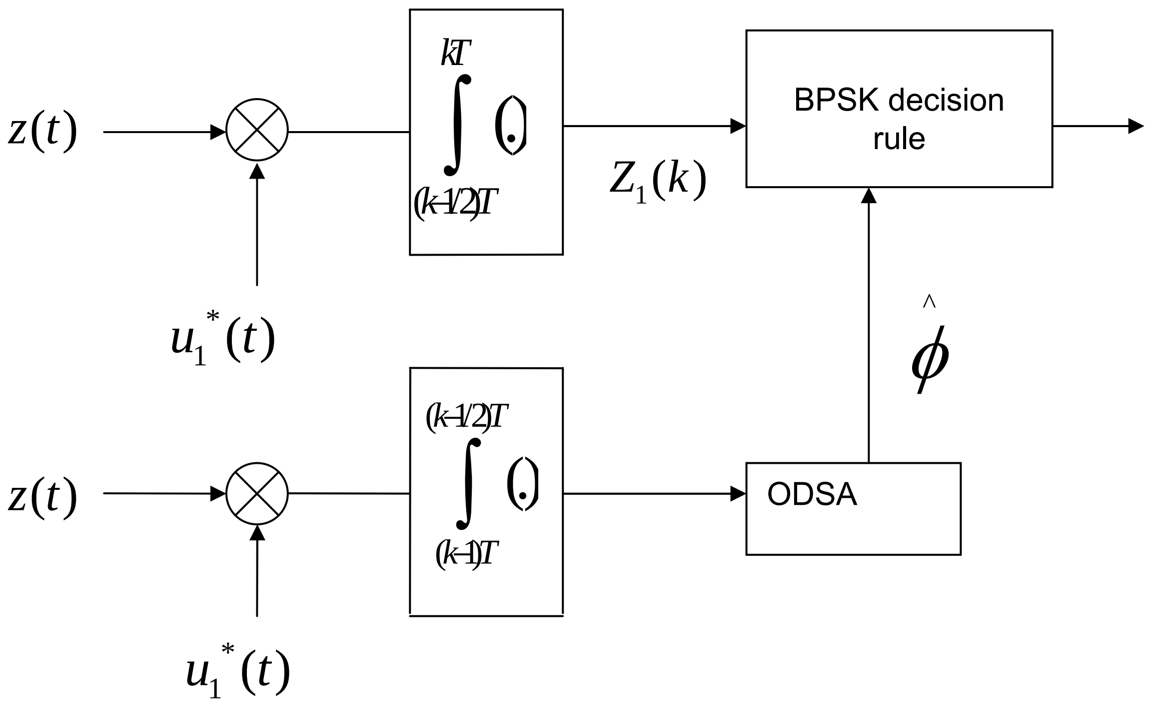
$$u_2(t) = \exp\left(+j\pi\frac{t}{T}\right)$$

BOS & HS-BPSK Signals





Half Symbol BPSK (HS-BPSK) Receiver



Values used in Simulation Results

$$SNR = E\left\{a^2\right\} \cdot E_b / N_0 = 2\gamma \cdot E_b / N_0 \qquad \gamma = 0.25$$ → Variance of Inphase and Quadrature components

$$\sigma^2 = 2EN_0 \quad for \quad Orthogonal \quad Signals$$

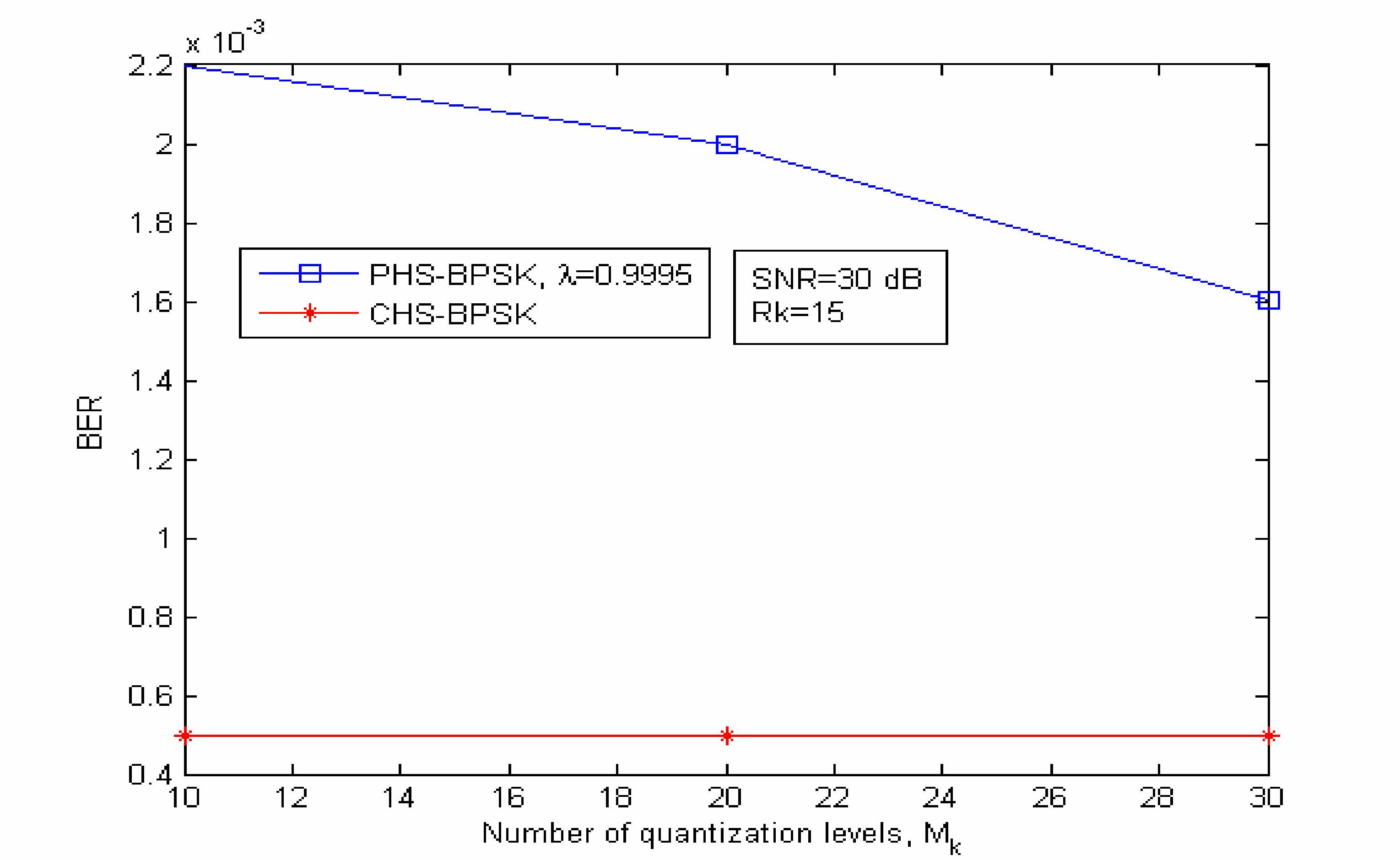
$$\sigma^2 = EN_0 \quad for \quad HS-BPSK \quad Signals$$

$$T = 10^5 \qquad E = T/2 \qquad \sigma_w{}^2 = 0.01 rad^2$$

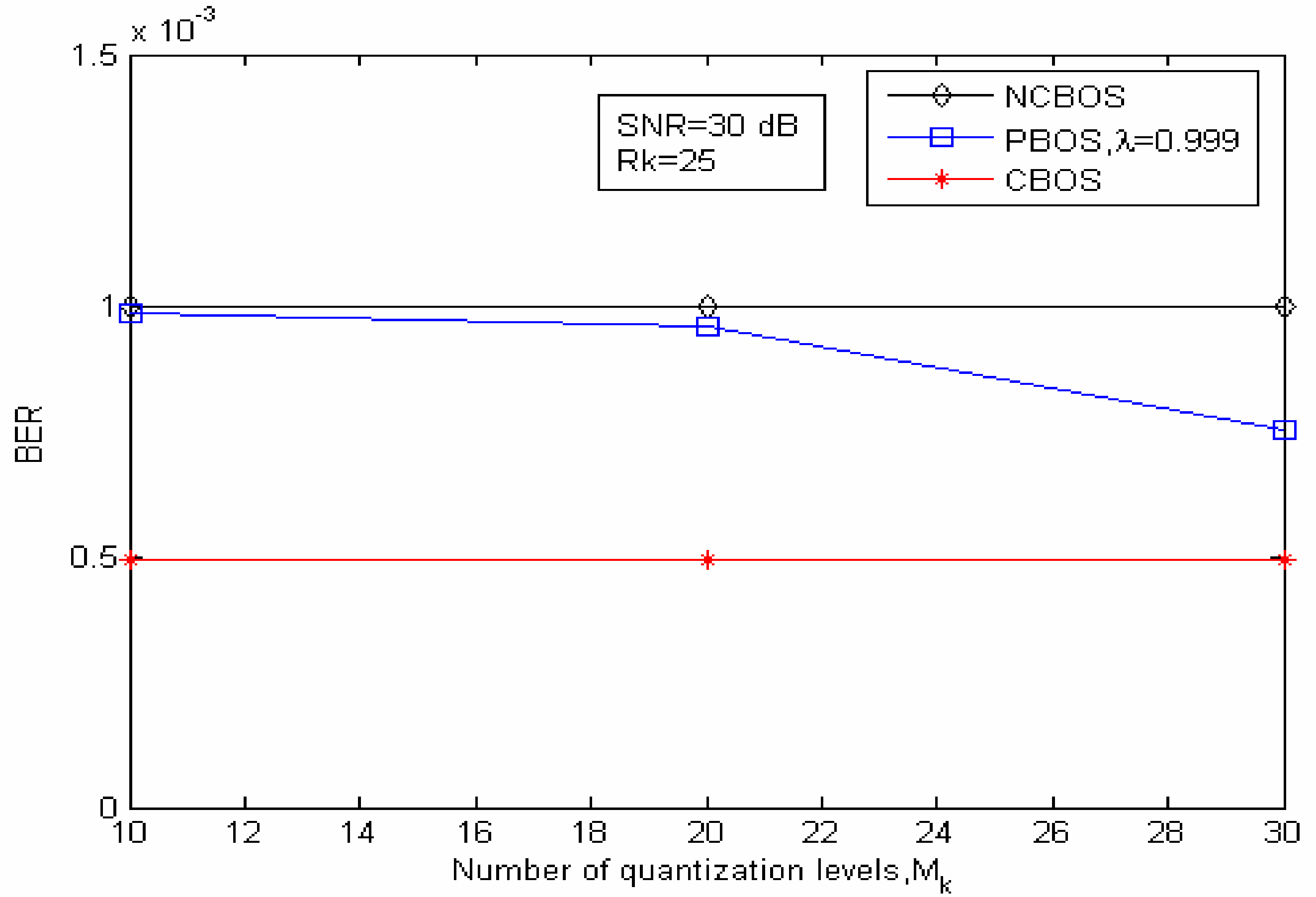
Signals are equally likely

Simulations with 1000 bits and 2000 runs

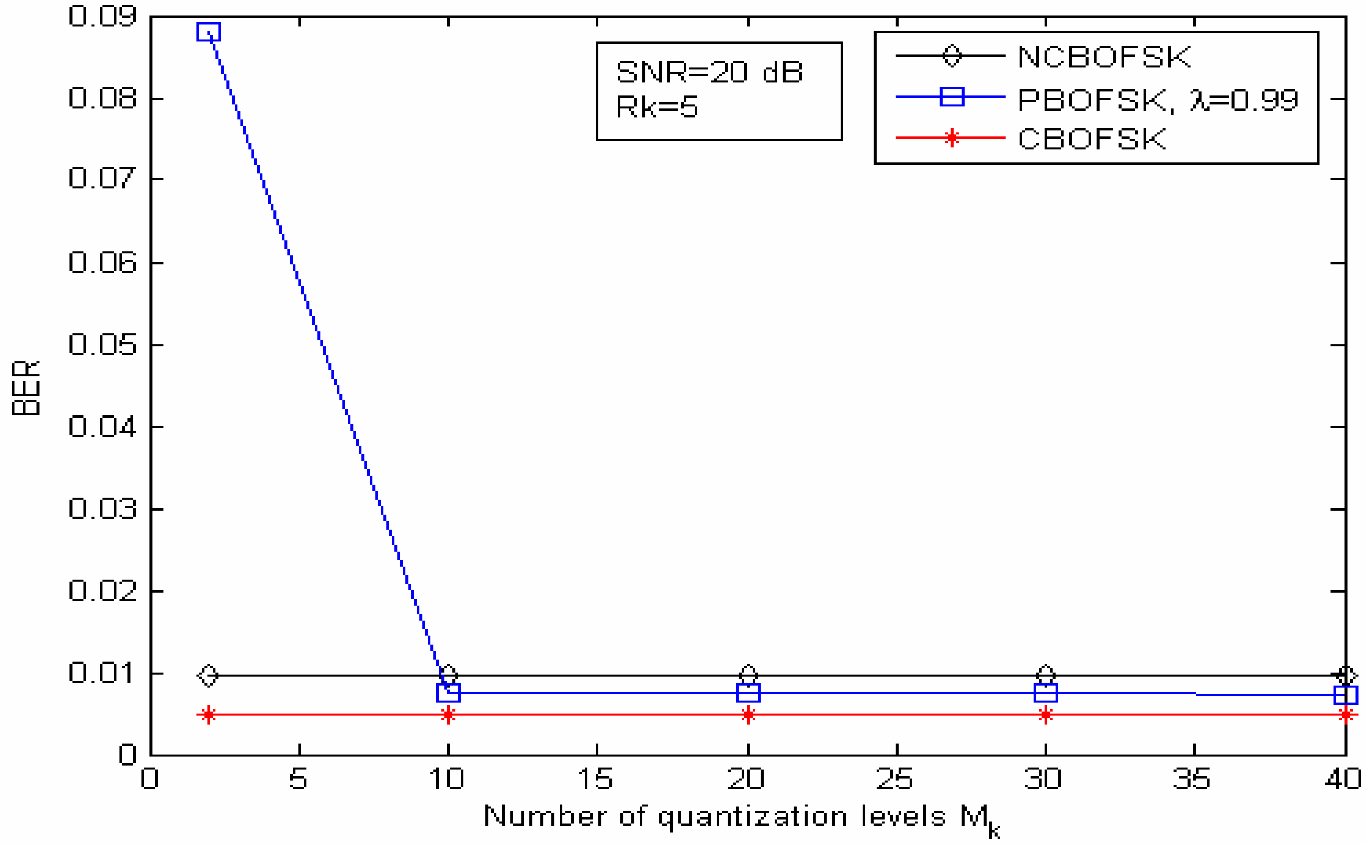
Performance comparisons of ODSA with increasing number of Quantization level or node, M_k (1/3)



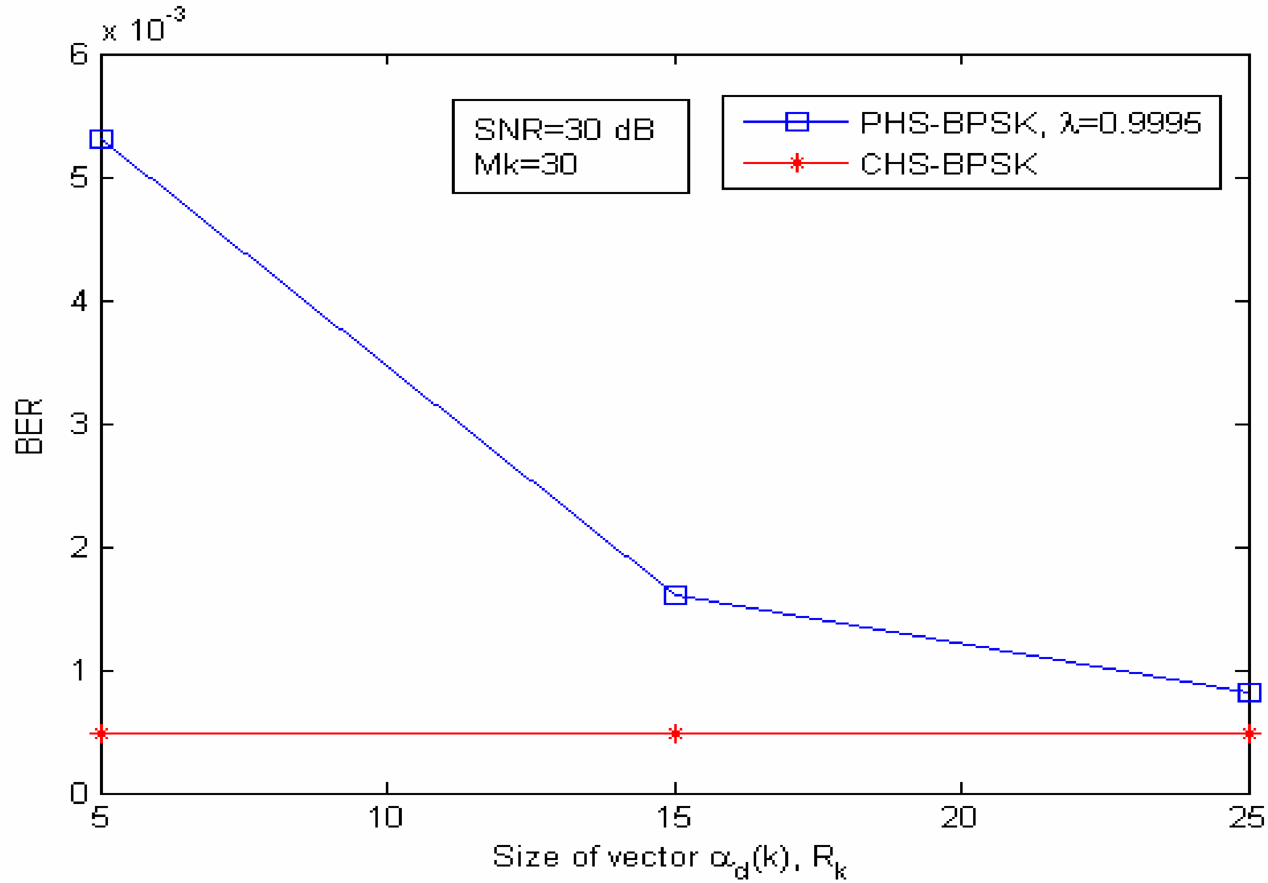
Performance comparisons of ODSA with increasing number of Quantization level or node, M_k (2/3)



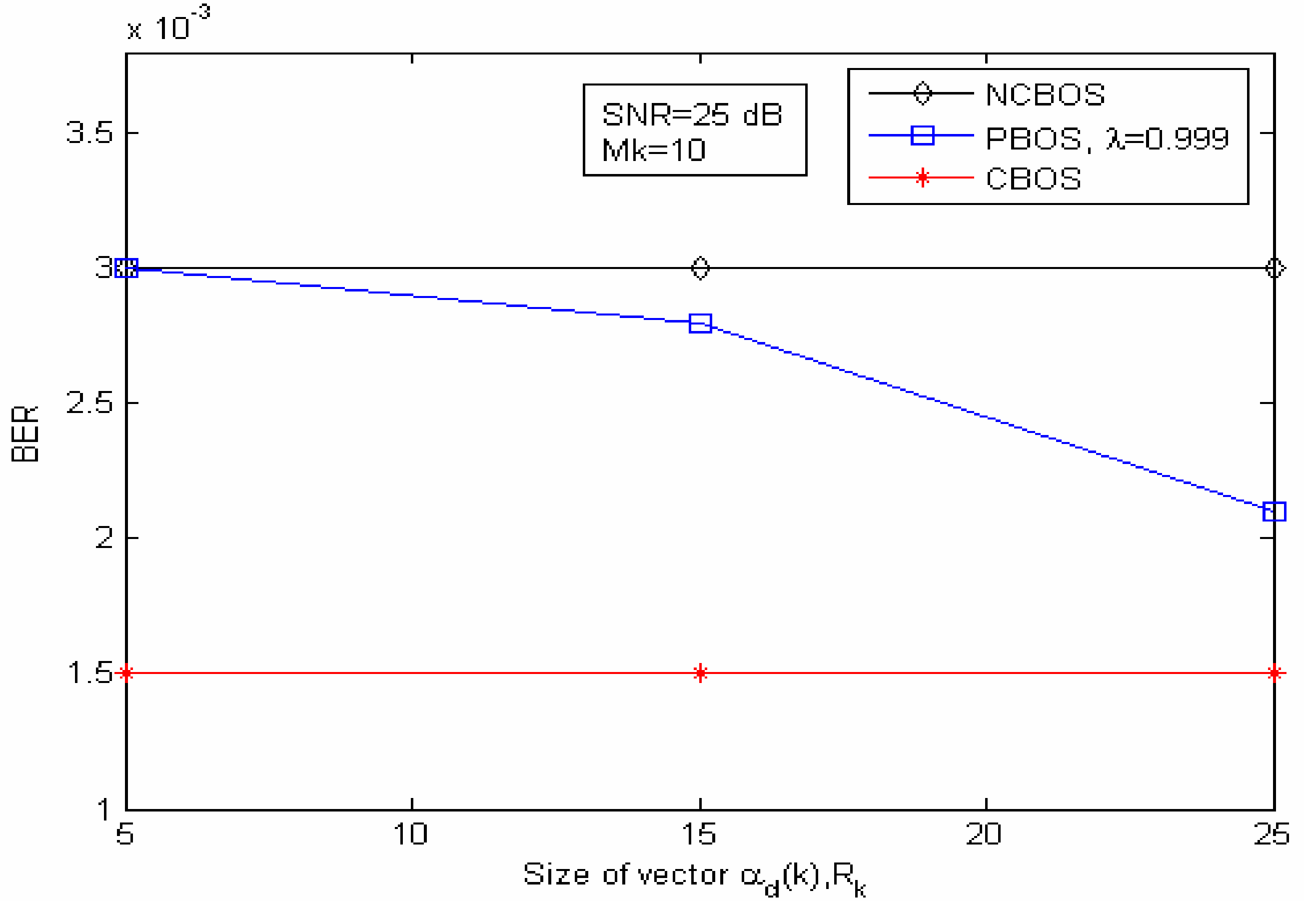
Performance comparisons of ODSA with increasing number of Quantization level or node, M_k (3/3)



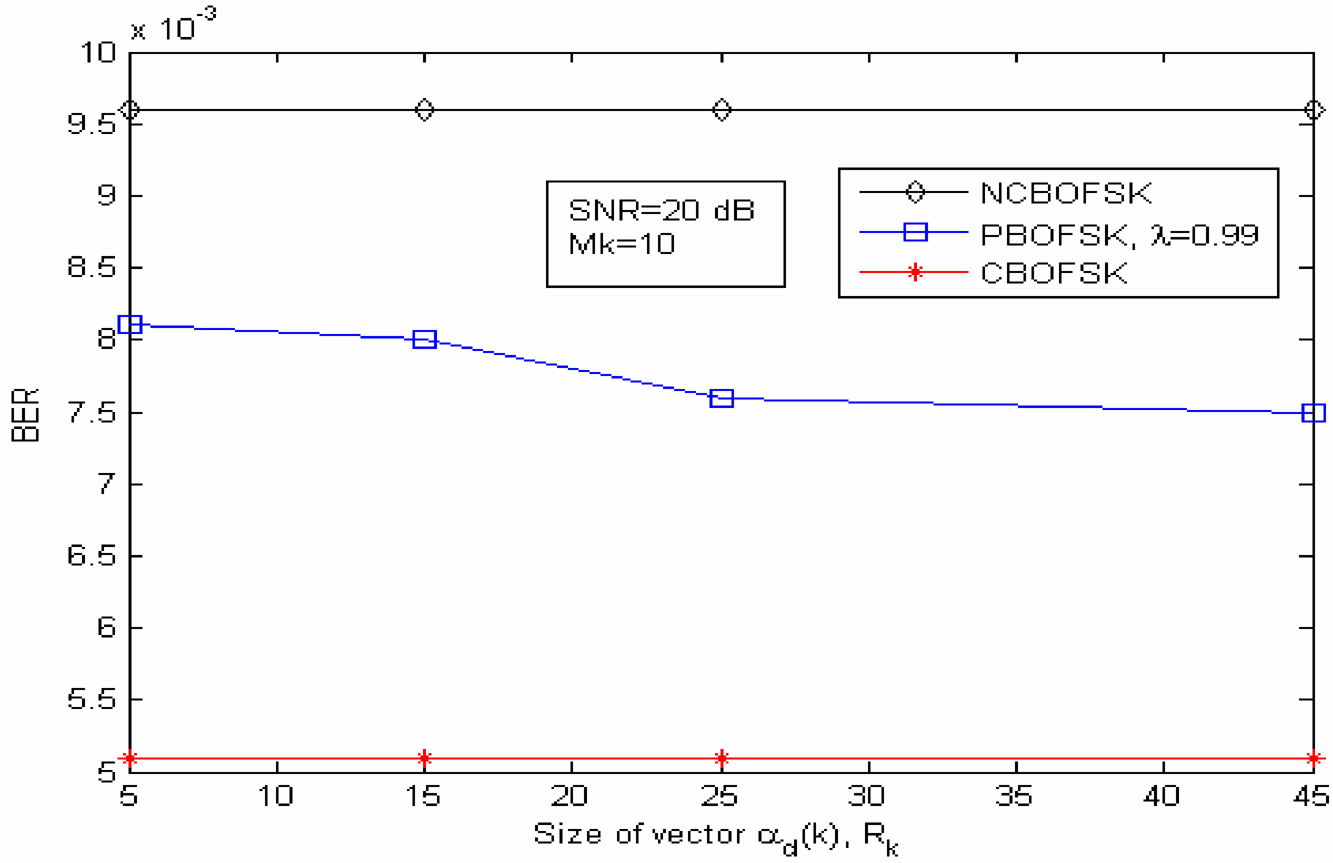
Performance comparisons of ODSA with increasing size of approximating discrete vector $\alpha_d(k)$, R_k (1/3)



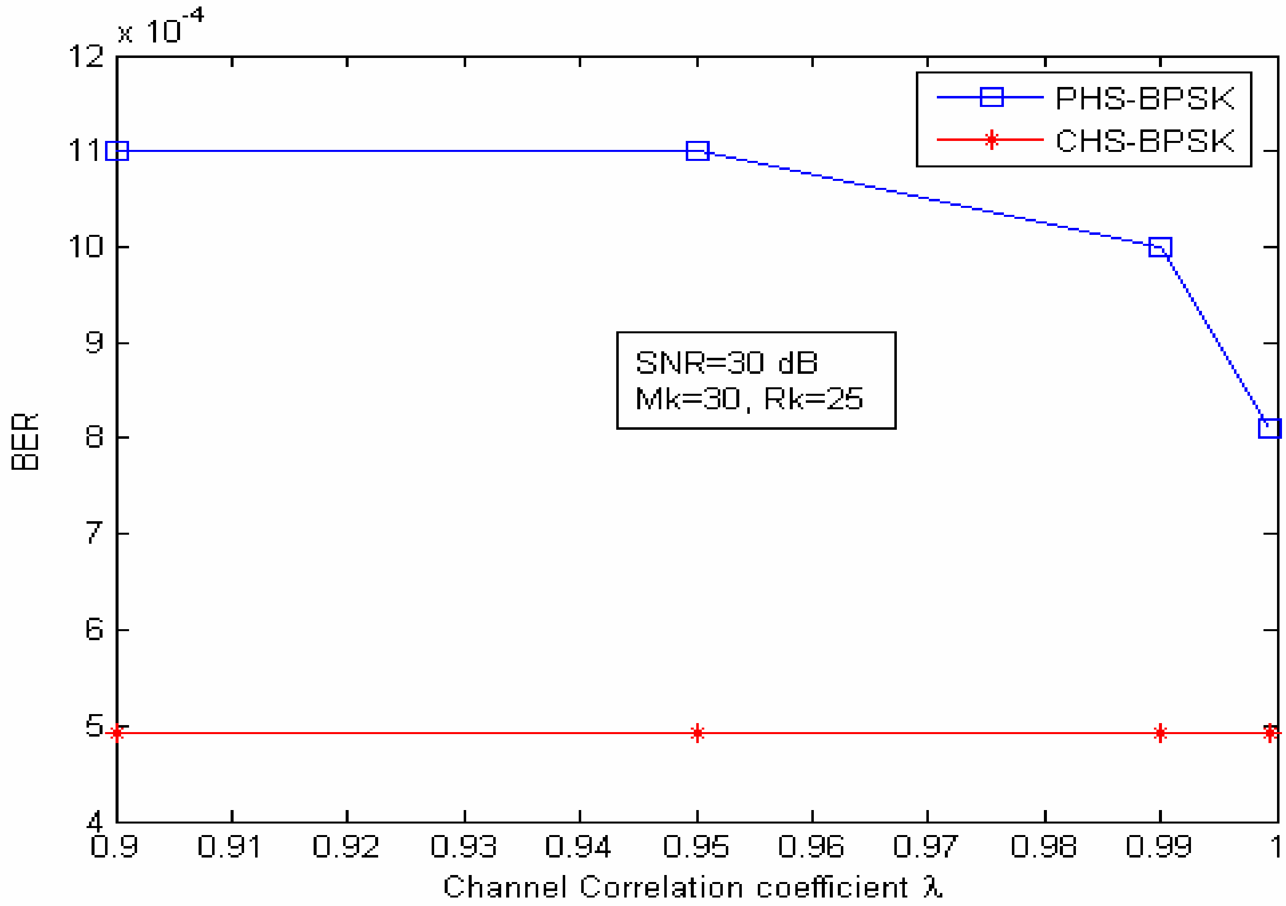
Performance comparisons of ODSA with increasing size of approximating discrete vector $\alpha_d(k)$, R_k (2/3)



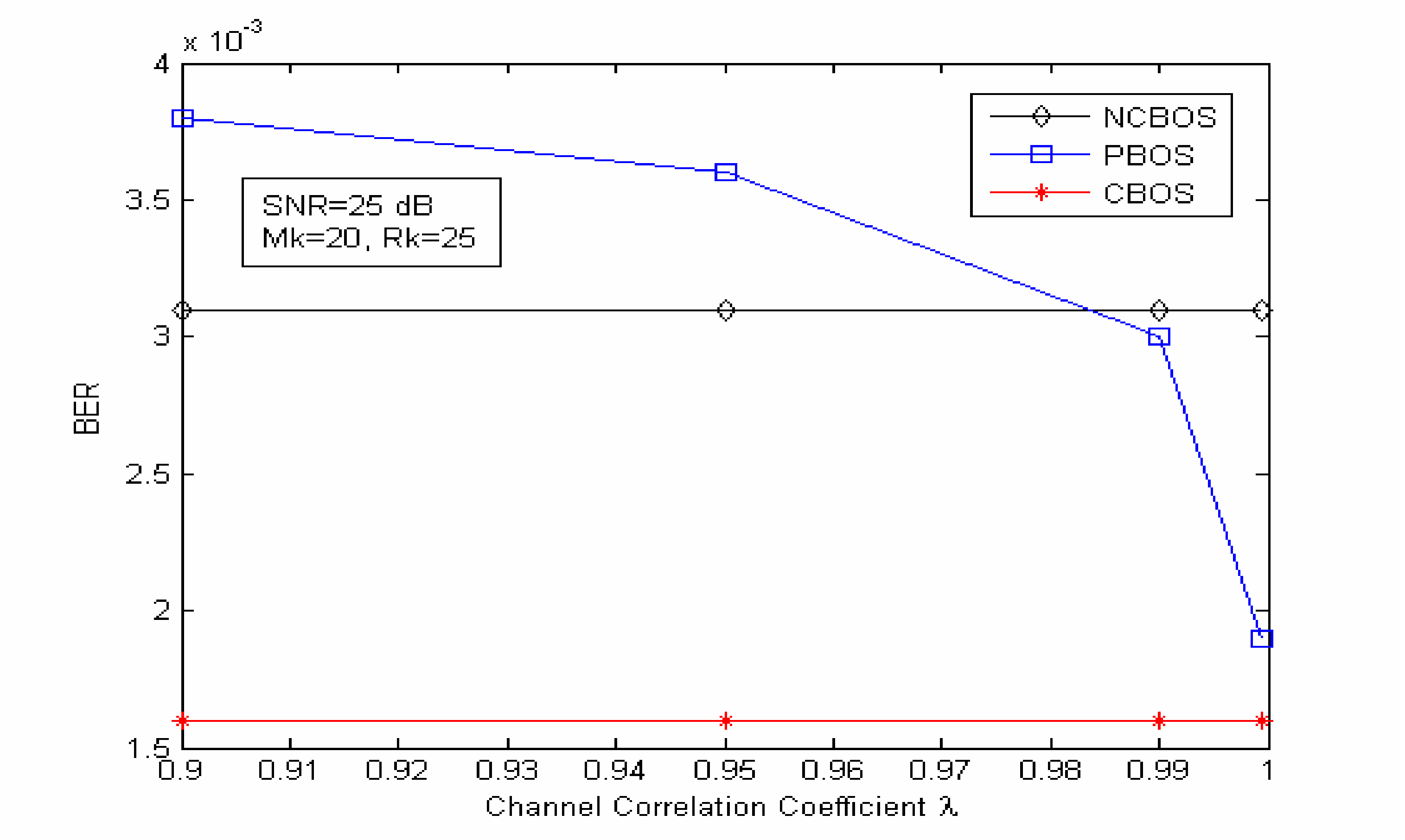
Performance comparisons of ODSA with increasing size of approximating discrete vector $\alpha_d(k)$, R_k (3/3)



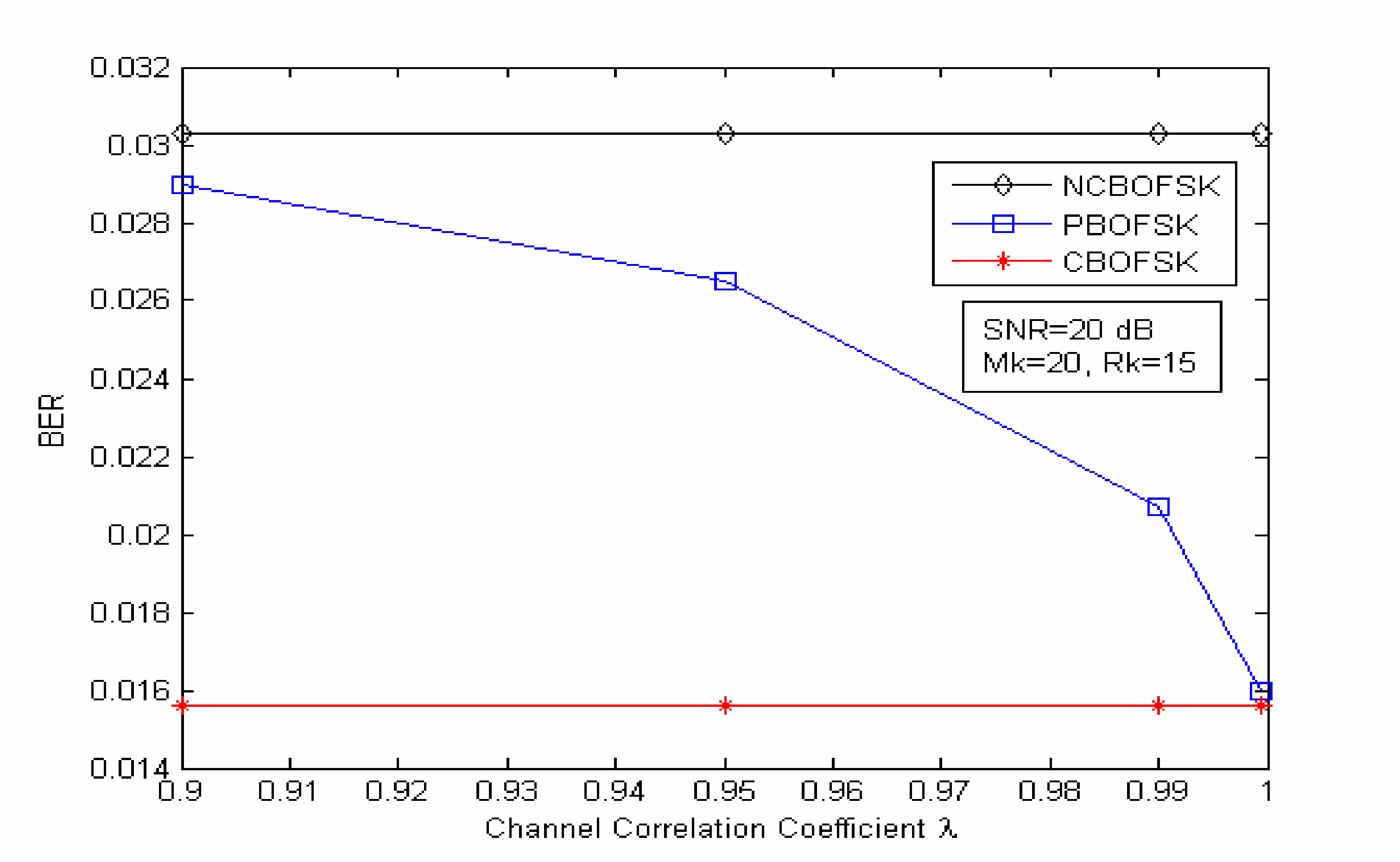
Performance comparisons of ODSA with different channel correlation coefficient, λ (1/3)



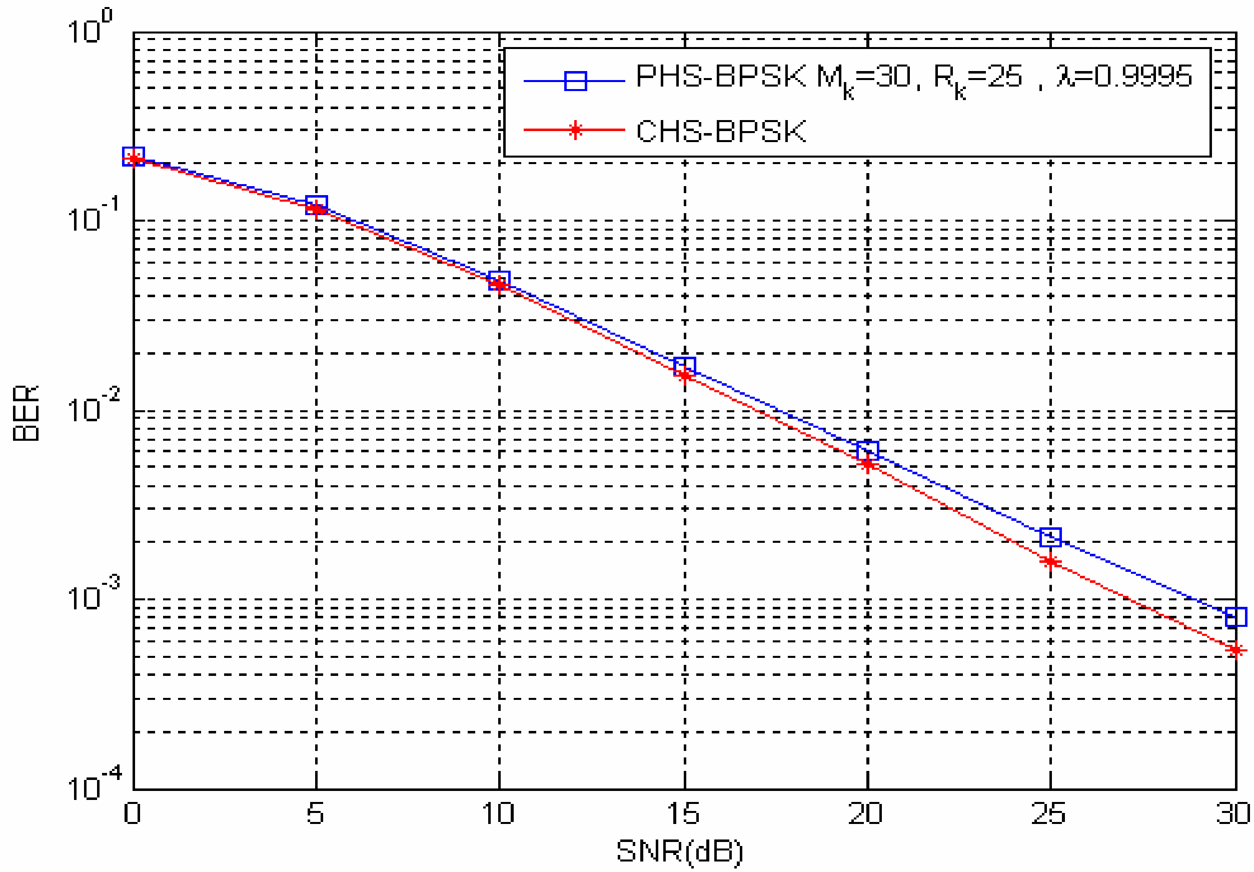
Performance comparisons of ODSA with different channel correlation coefficient, λ (2/3)



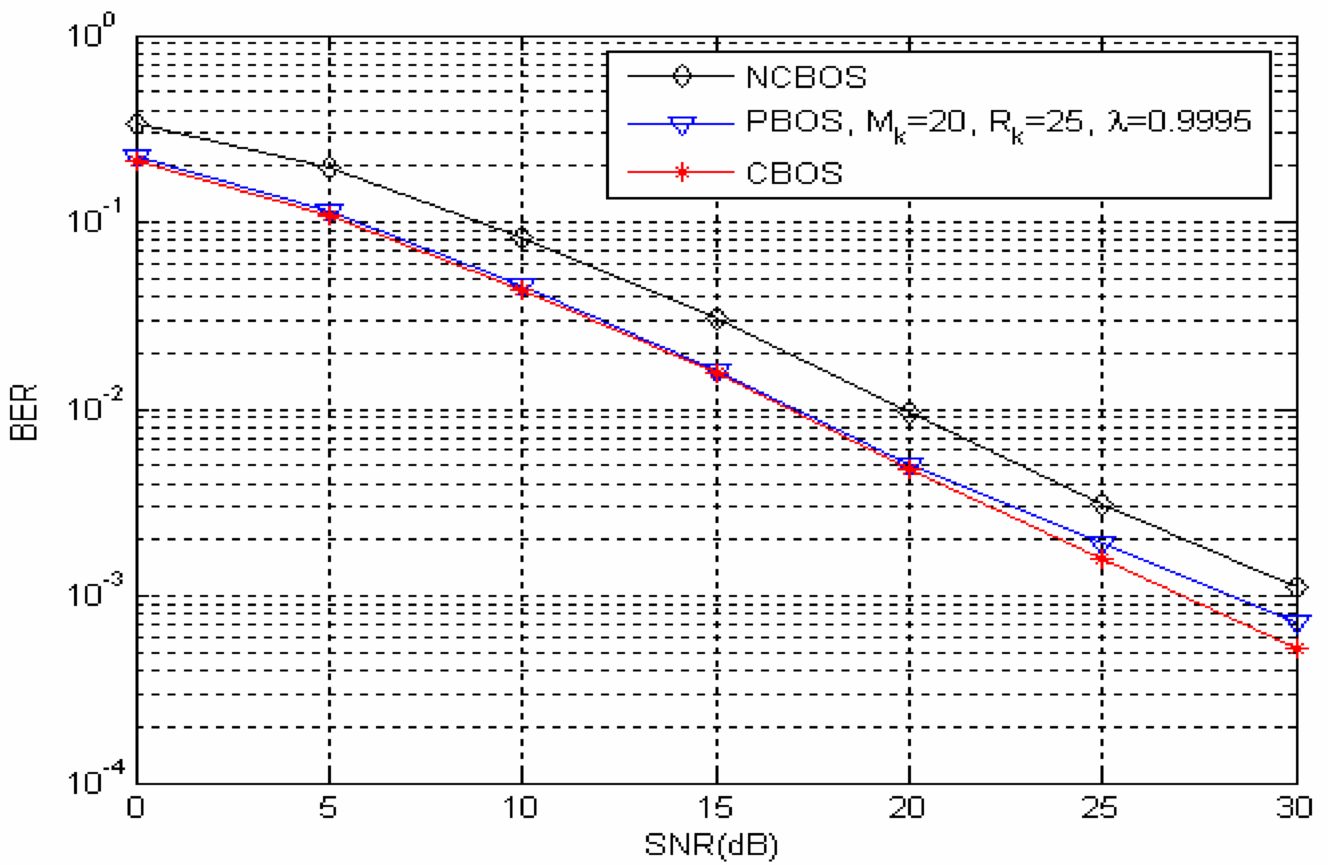
Performance comparisons of ODSA with different channel correlation coefficient, λ (3/3)



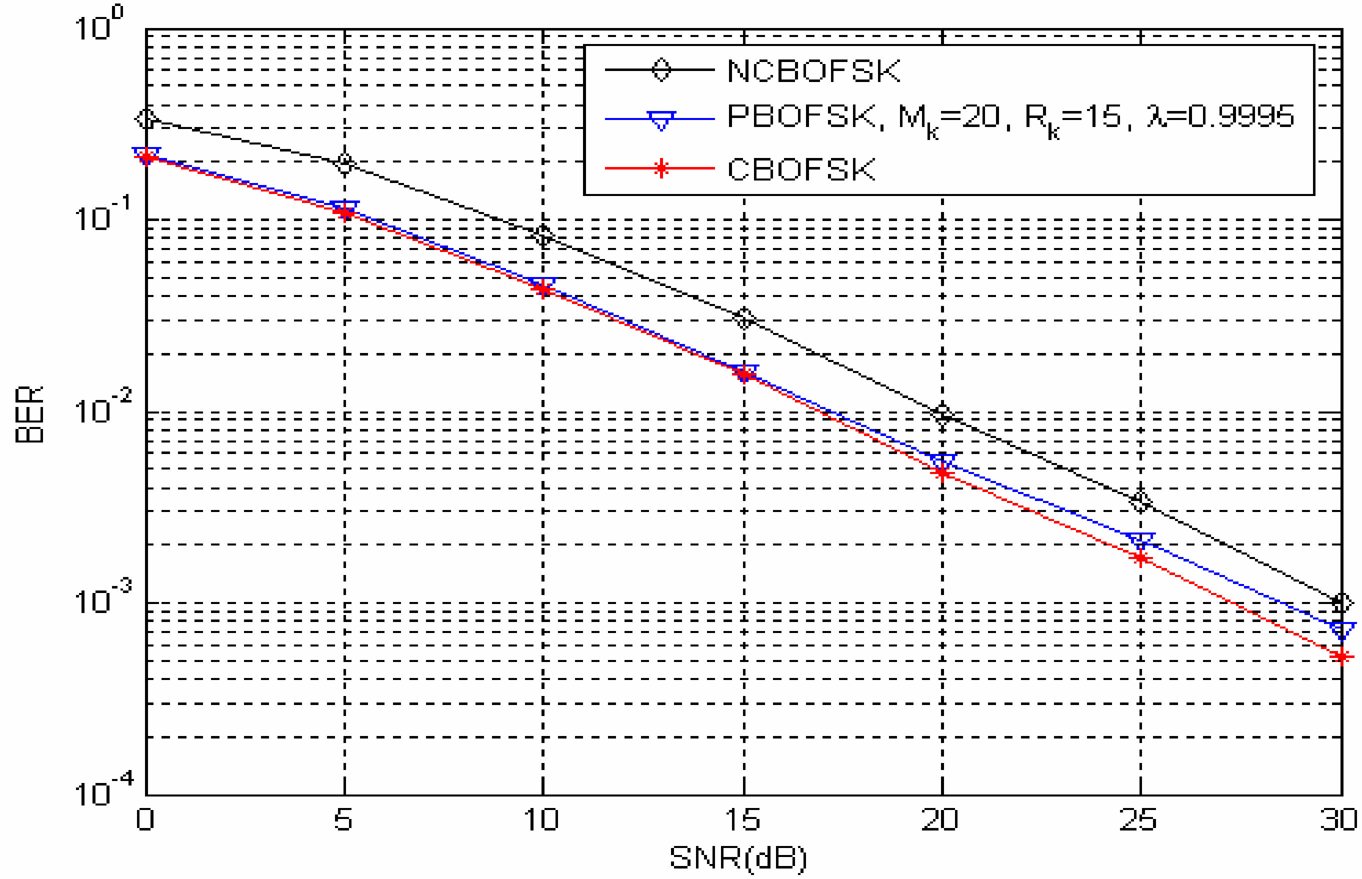
Performance comparisons of ODSA with different SNR (1/3)



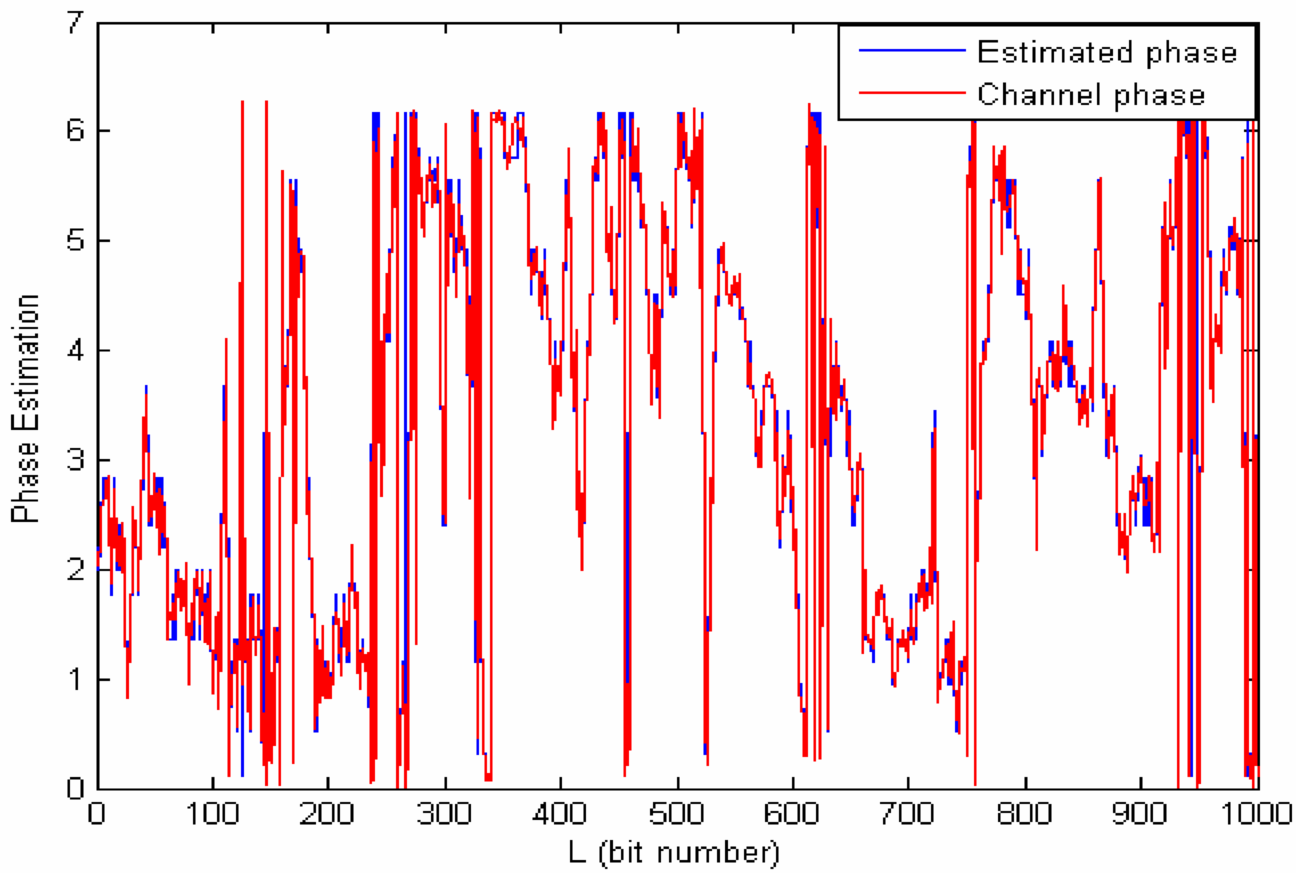
Performance comparisons of ODSA with different SNR (2/3)



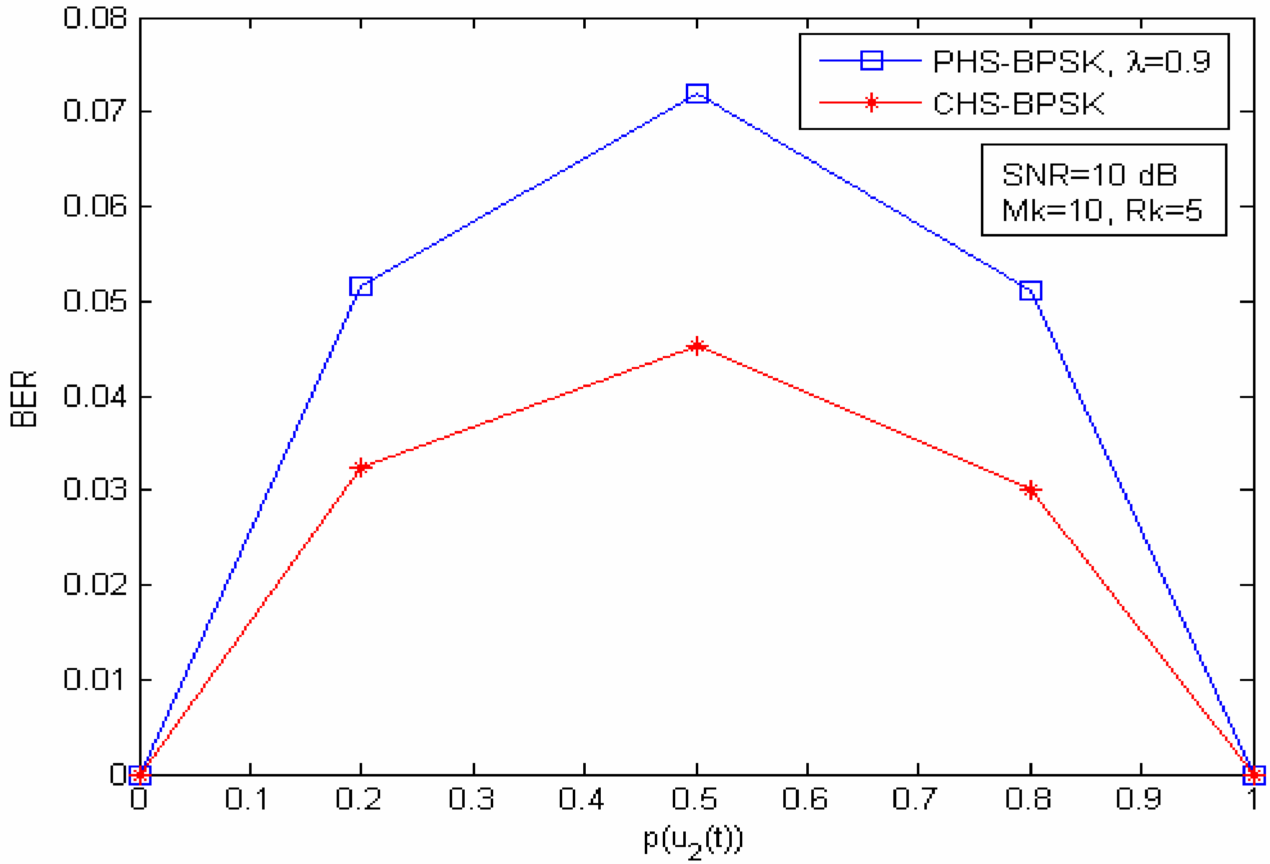
Performance comparisons of ODSA with different SNR (3/3)



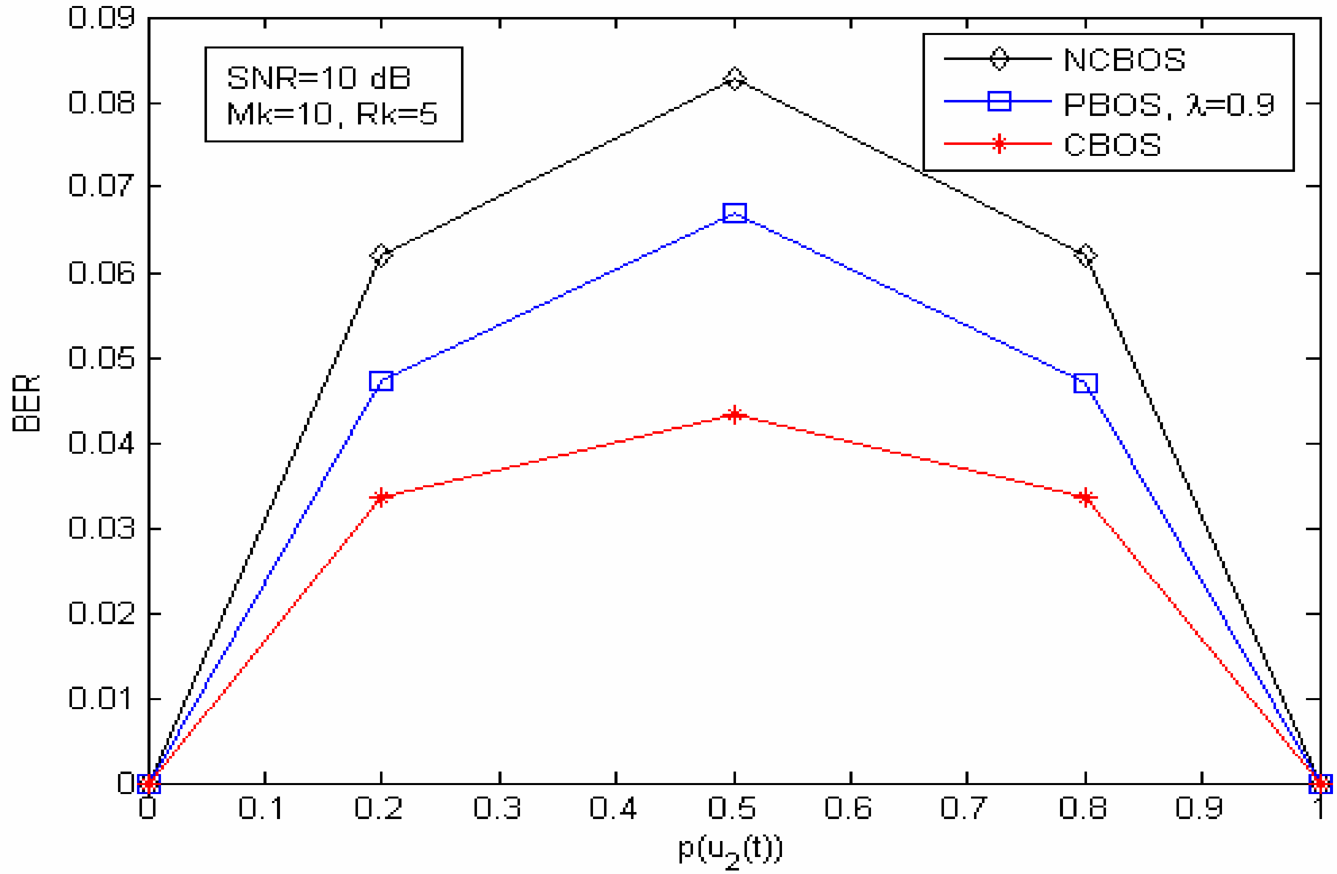
Channel Phase v.s. Estimated Phase



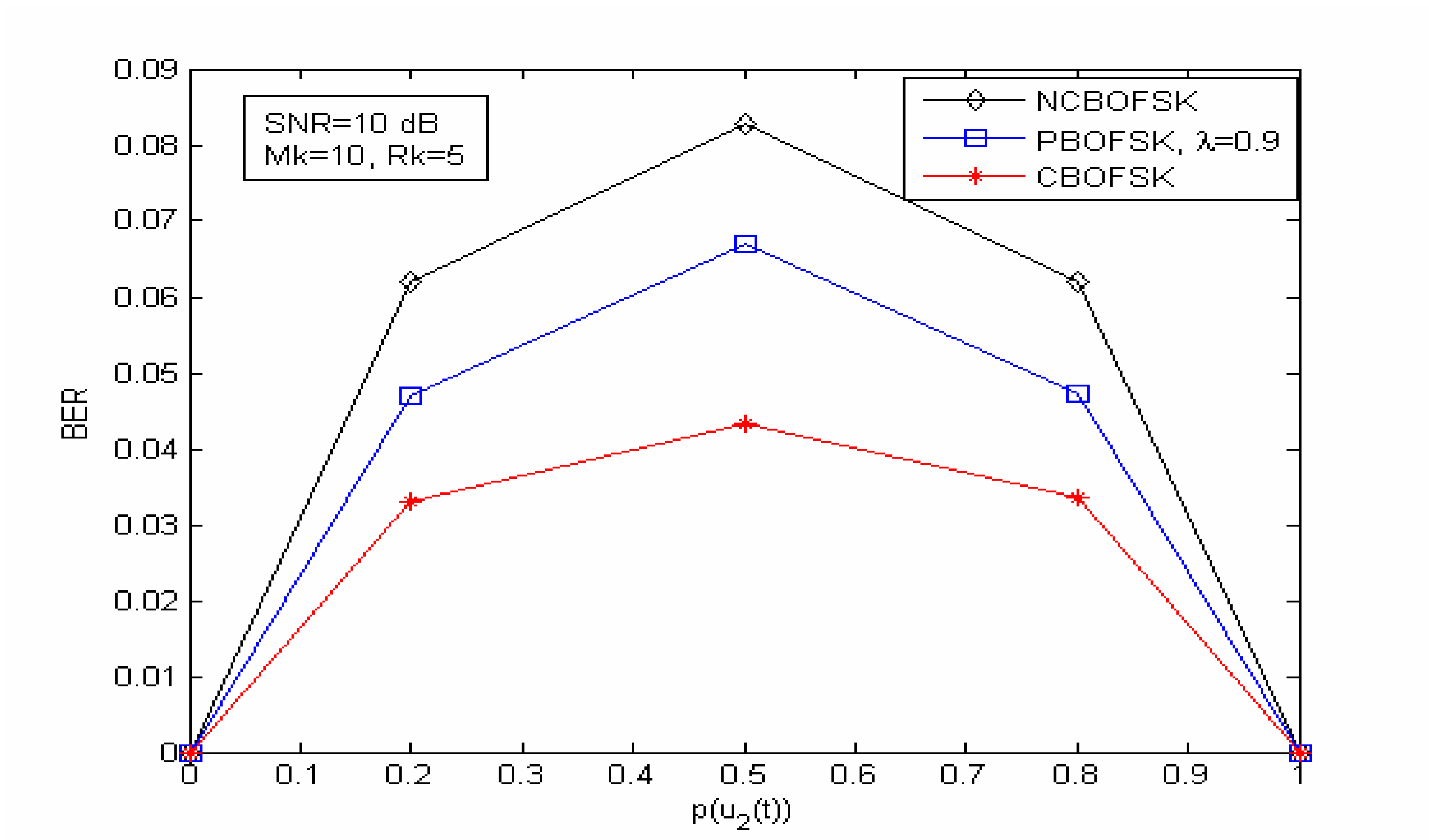
Apriori Probability of u2(t) v.s. BER (1/3)



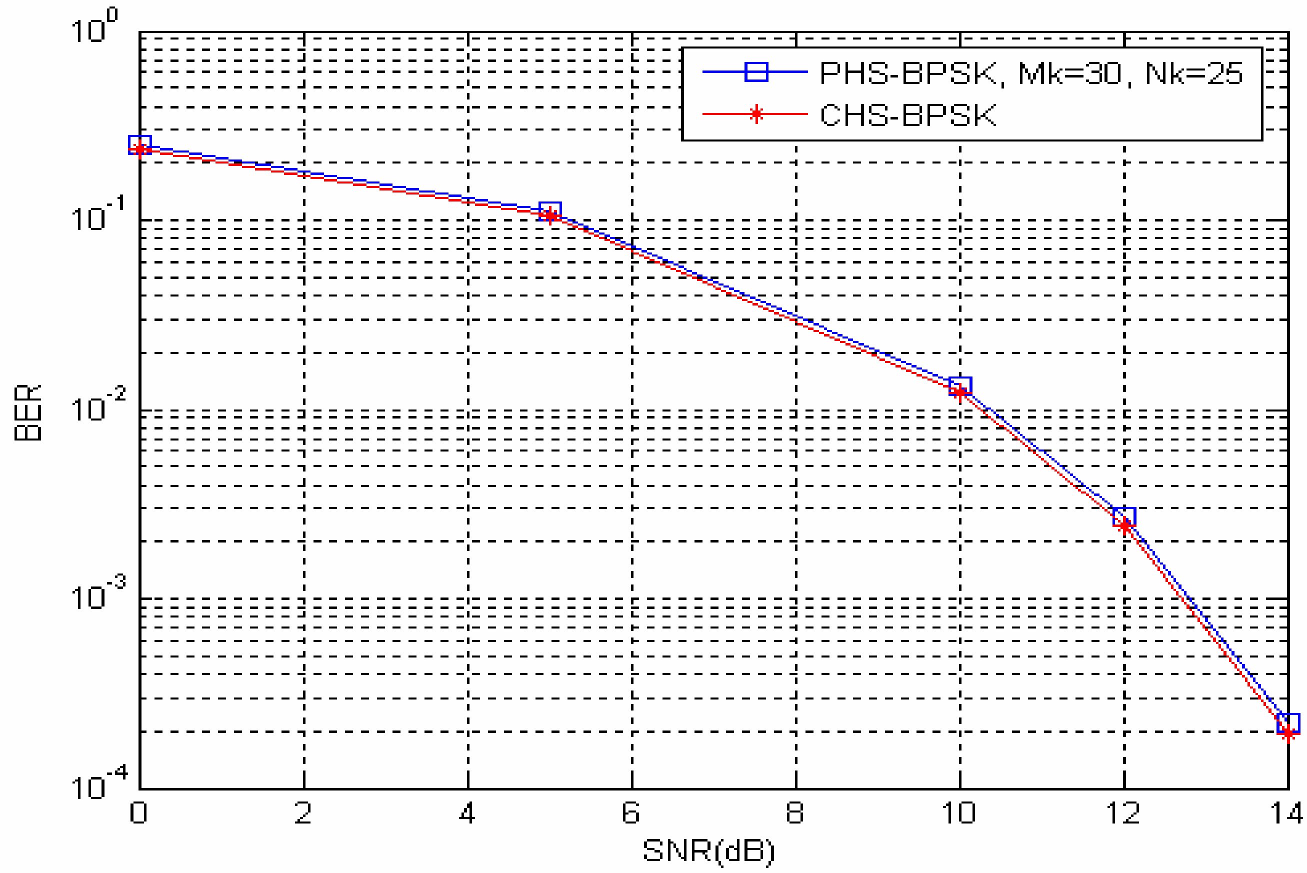
Apriori Probability of u2(t) v.s. BER (2/3)



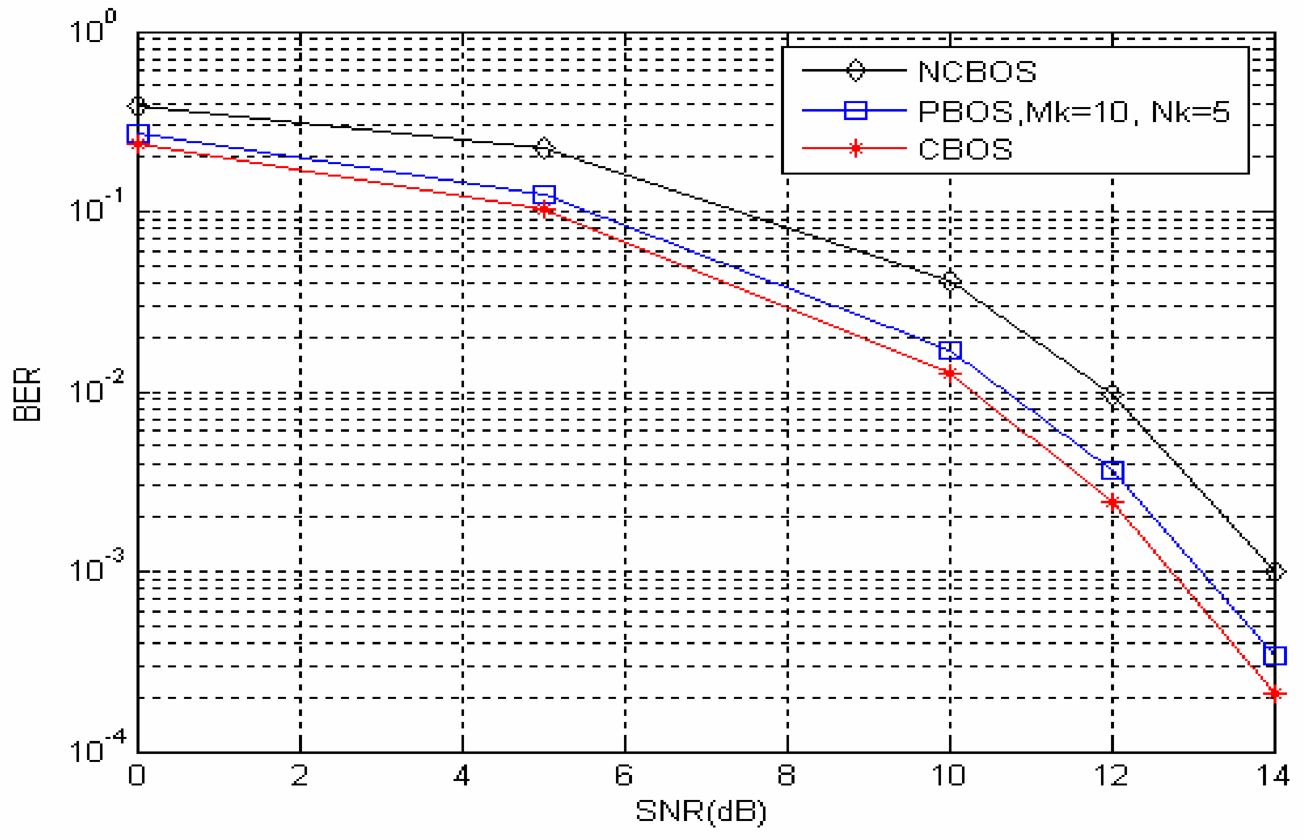
Apriori Probability of u2(t) v.s. BER (3/3)



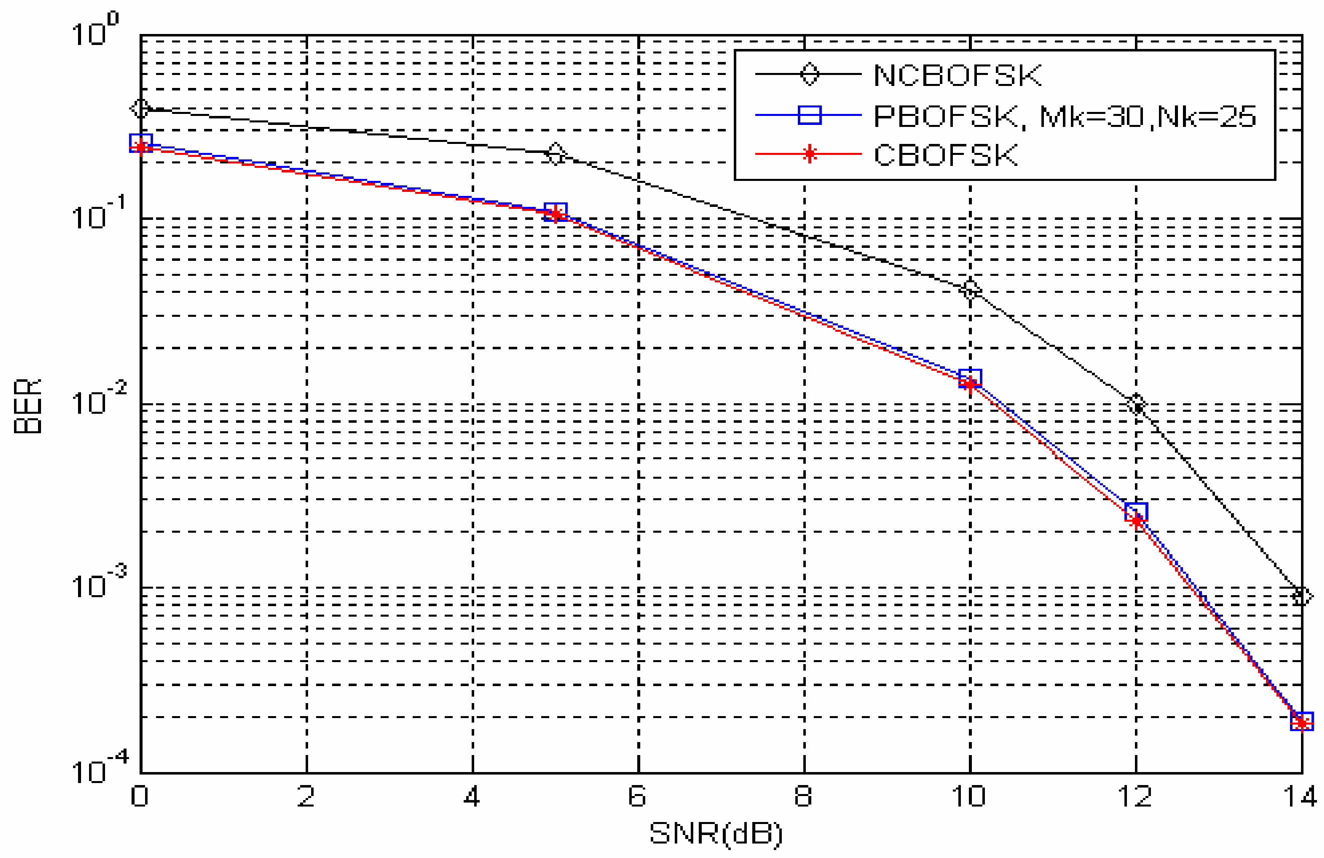
Gaussian Random Walk Phase Transition Model (1/3)



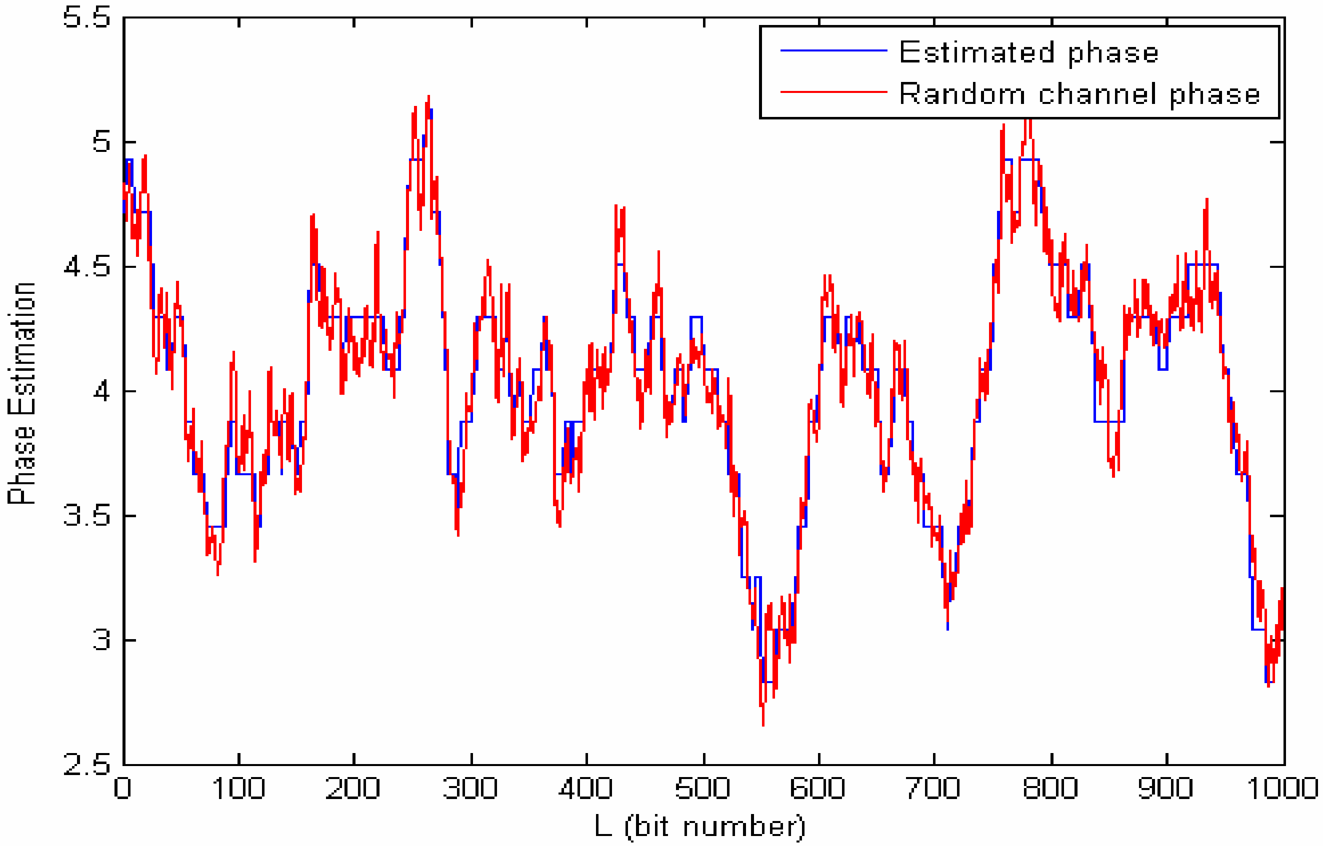
Gaussian Random Walk Phase Transition Model (2/3)



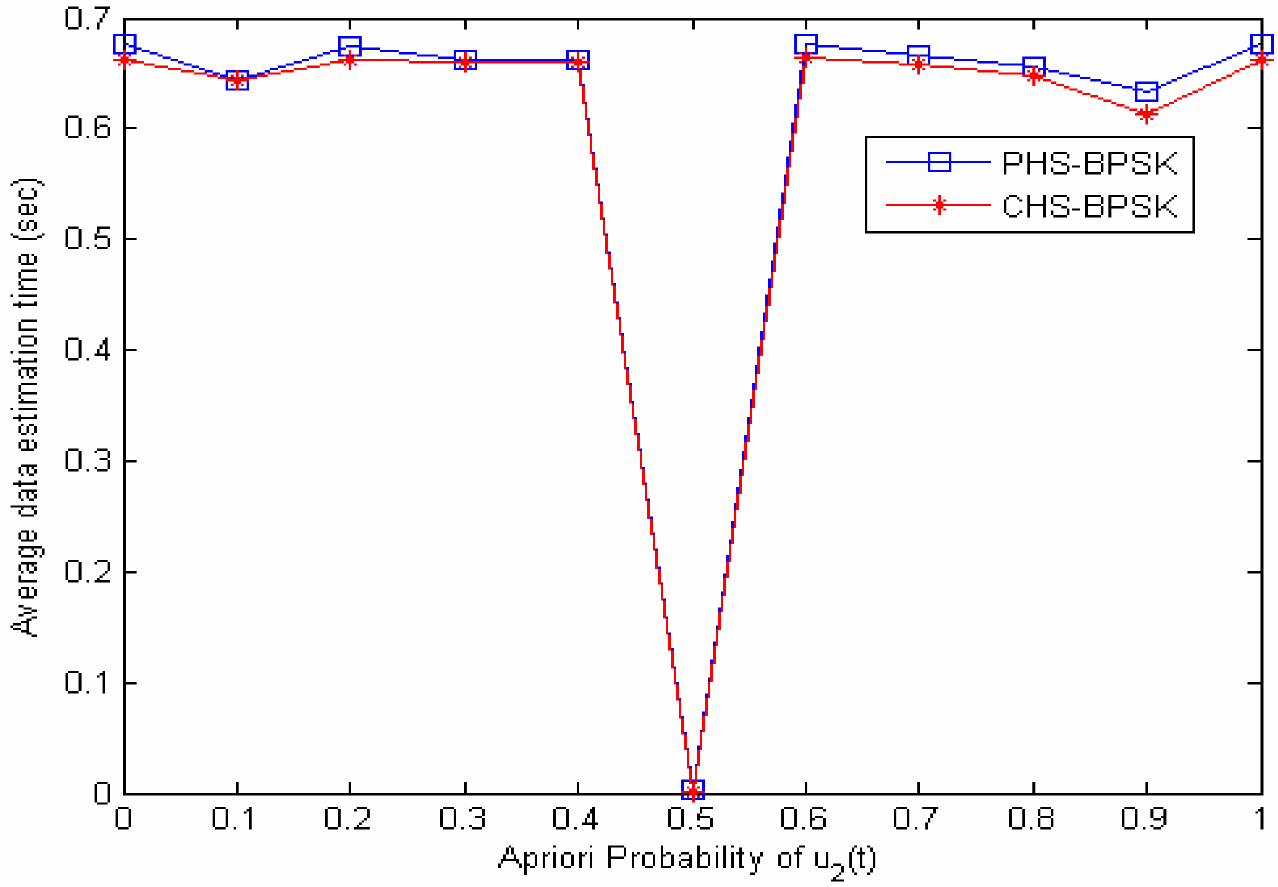
Gaussian Random Walk Phase Transition Model (3/3)



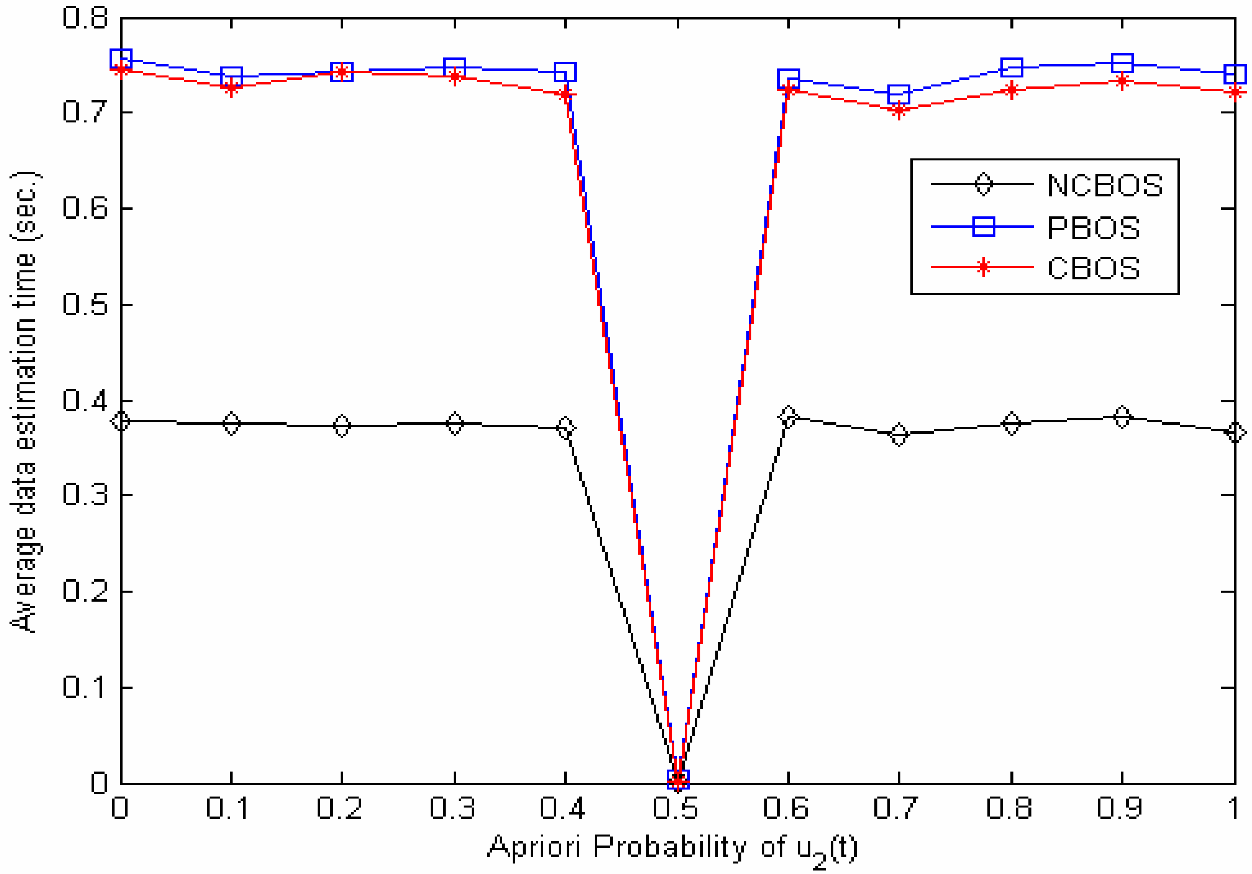
Channel Phase v.s. Estimated Phase



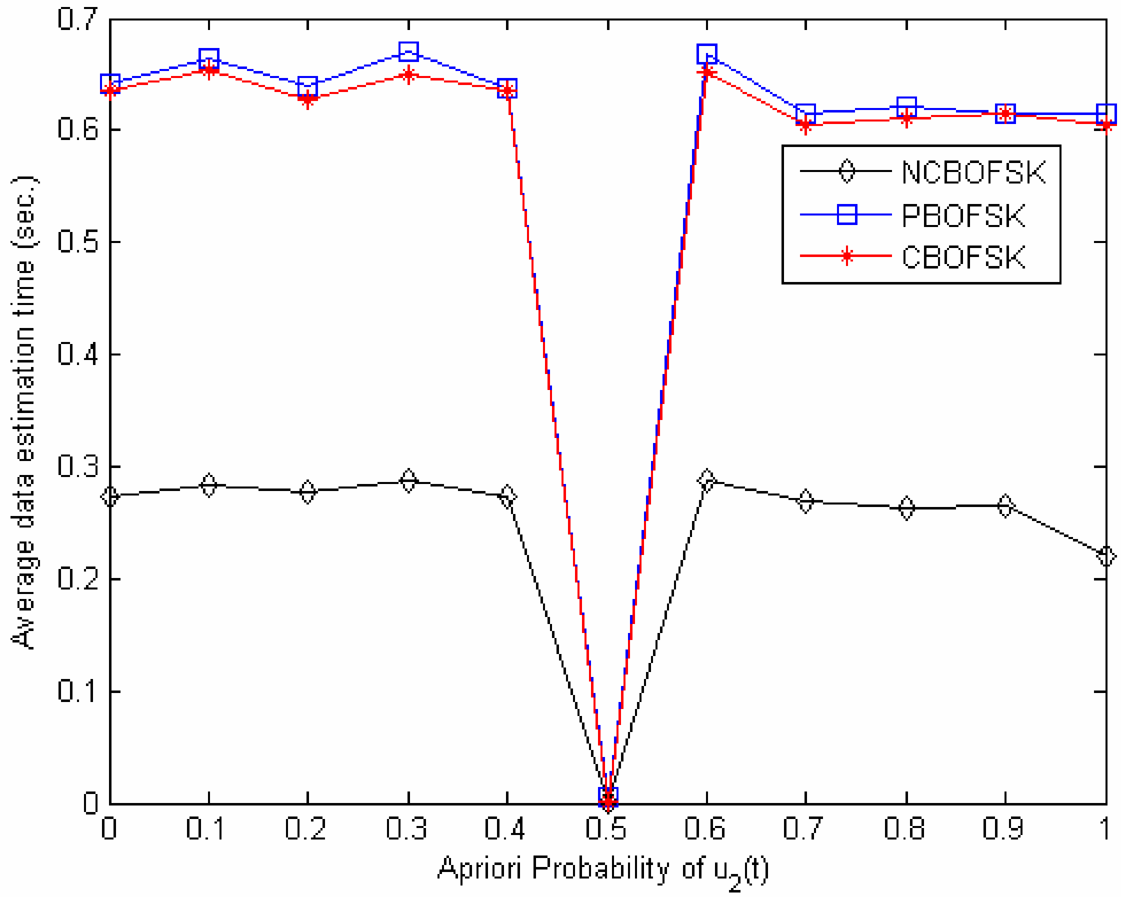
Computational time of data estimation (1/7)



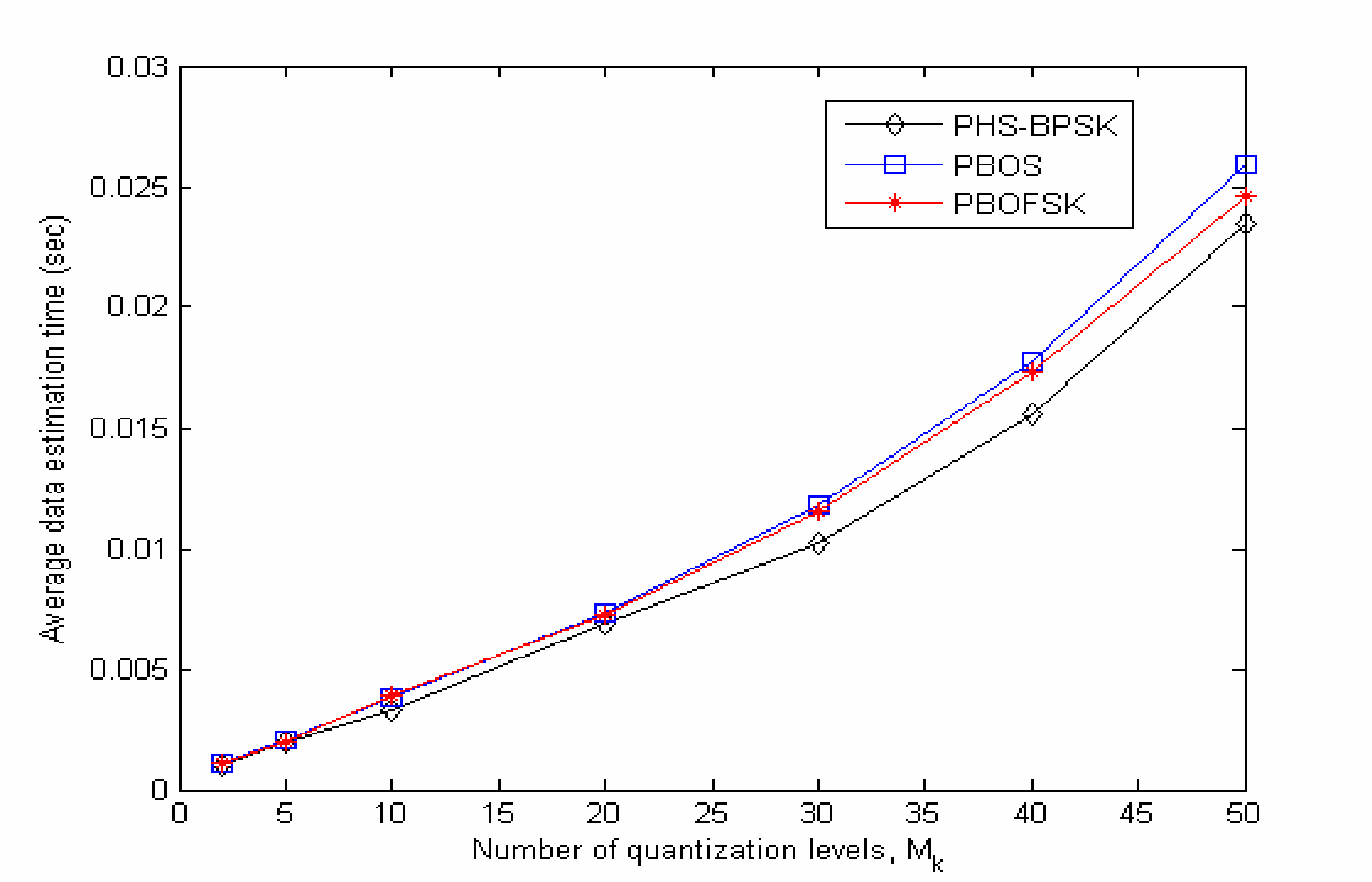
Computational time of data estimation (2/7)



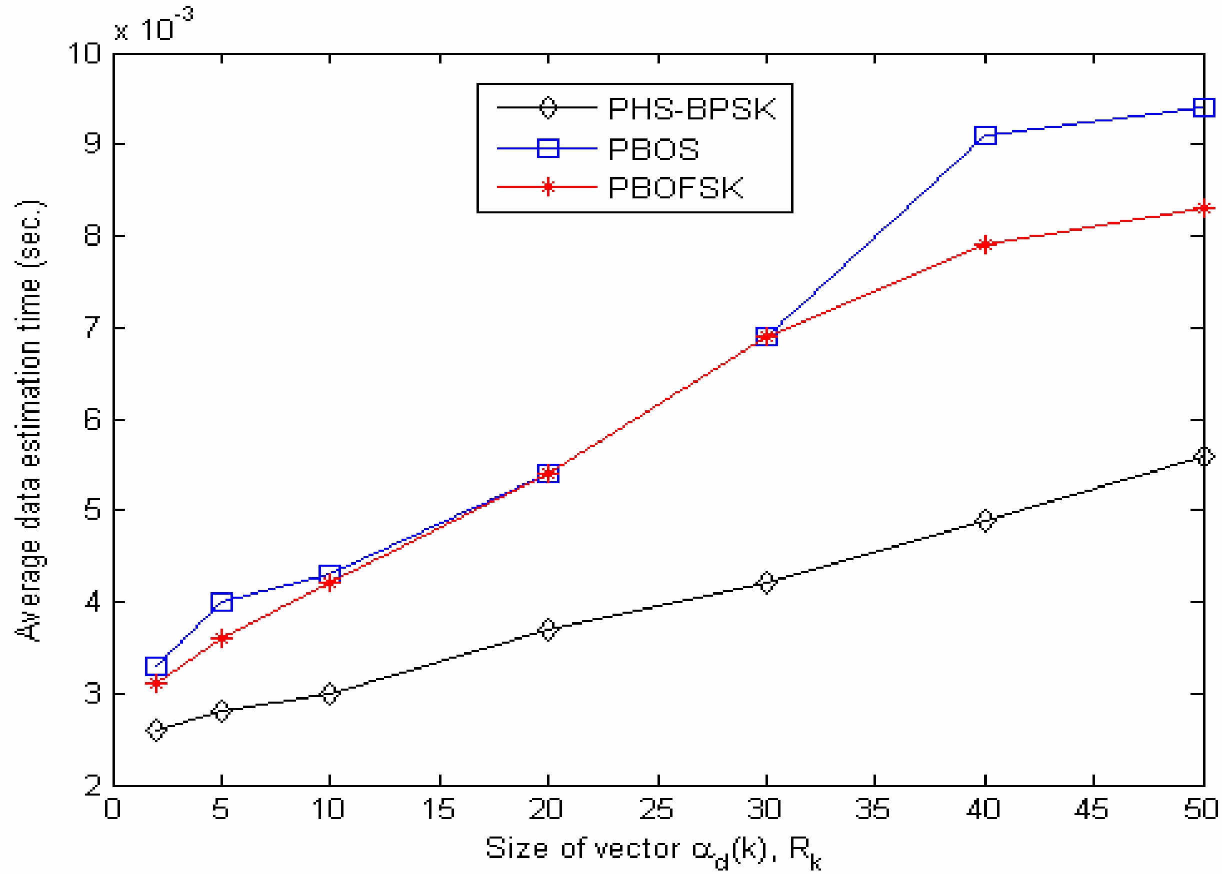
Computational time of data estimation (3/7)



Computational time of data estimation (5/7)



Computational time of data estimation (6/7)



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Conclusion

- ODSA can be useful for robust phase estimation in fading channels.
- One drawback may be the computational time of the algorithm.
- Another drawback is that we have to know apriori statistical properties of the channel.

Future Work

- Channel Phase Estimation with ODSA can be extended for M-ary FSK and M-ary PSK signals.
- Complex Gaussian channel coefficient estimation with Kalman Filtering by transition of coefficient model under uncertain observations.
- Performance of ODSA can be compared with phase tracking loops like Costas loop.

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THANK YOU