

# Differential Space-Frequency Group Codes for MIMO-OFDM

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**Abstract**— In recent years, *coherent* space-frequency (SF) coded multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems have attracted great attention. However, obtaining perfect channel estimates for frequency-selective channels with lots of taps is challenging and computationally complex. In this paper, we present a transmission scheme that uses *differential space-frequency group codes* (DSFC) for MIMO-OFDM systems. Our proposed scheme is based on the theory of space-frequency group codes. We use the group properties of space-frequency codes that has recently been investigated and obtain a differential transmission rule incorporated with subcarrier allocation method that allows data to be sent without channel estimates at the transmitter or receiver. Recently proposed noncoherent space-frequency codes using MIMO-OFDM have exponential computational complexity with code rate in frequency-selective channels. Compared to existing noncoherent space-frequency codes, our proposed code design is computationally more efficient and is easier to implement. We also demonstrate that the presence of additional resolvable channel taps does not monotonically improve the performance of DSFCs, which shows that obtaining full space-frequency diversity order for increasing number of channel taps decreases the performance of the receivers with lack of channel state information (CSI). This result is completely different from the coherent case of space frequency group codes in which additional channel taps increase the BER performance due to the exploitation of frequency diversity present in MIMO-OFDM frequency-selective channels.

**Index Terms**— Differential modulation, space-frequency group codes, MIMO-OFDM, frequency-selective fading channels.

## I. INTRODUCTION

Space-Frequency coding has emerged as a promising technique for realizing spatial and frequency diversity gains in multiple-input multiple-output (MIMO) wireless systems [1],[2],[6],[8],[10]. Most research on space-frequency coding has been done on assuming availability of perfect channel fading estimates at the receiver. This is reasonable if the channel changes slowly compared with the fading rate, since the transmitter can send training symbols or pilot tones to enable the estimation of channel fading parameters on each subcarrier in MIMO-OFDM systems.

In some situations like rapid fading conditions or increasing number antennas, it is difficult to obtain channel estimation or channel state information (CSI) when reducing the cost and complexity of the receiver is our main objective. For example, in frequency hopping systems, fading conditions may change significantly from one hop to the next or in

high-mobility conditions, it is not clear whether accurate channel estimation is possible [4]. For such situations, we need to develop efficient coding or modulation schemes that do not require channel estimates at the transmitter or receiver. Frequency shift keying (FSK) and differential phase shift keying (DPSK) can be demodulated without channel estimates at the receiver for a single transmit antenna. For space-time and space-frequency diversity in multiple transmit antennas, differential and noncoherent schemes were proposed in [2],[3],[4],[8],[12],[13],[14],[16],[15] and [17].

Noncoherent MIMO-OFDM systems are analyzed in [2] and [12]. Noncoherent space frequency codes derived in [2] are not particular to differential transmission scheme and are exponential in the code rate. Therefore, if the the data rate is high, implementing those codes requires complex algorithms. Differential space-time modulation with transmit diversity based on group codes were designed in [4]. The group property of theses codes makes it easier to analyze the structures, and yields simpler modulation and demodulation schemes. In [13], spatial diversity achieving differential space-time coding systems were proposed but this scheme does not exploit the additional degrees of freedom offered by multipath propagation i.e. discards the frequency diversity which is available in the wideband frequency-selective fading channels. Schemes that achieve both spatial and frequency diversity in MIMO-OFDM for differential encoding and decoding are investigated in [14],[15],[16] and [17]. Differential schemes in the frequency domain was proposed in [8] and [16] where the differential encoding/decoding processes were performed over adjacent subcarriers within each OFDM symbol. However in [8], designing specific differential space frequency code construction method according to pairwise error probability analysis is not followed and in [16], the subcarrier grouping is not chosen according to channel delay profile of the frequency-selective channels.

In this paper, we want to extend the design approach followed in [1] for coherent reception to differential case when the channel estimates are not available at the transmitter and receiver. We propose *differential space-frequency group codes* (DSFC) for MIMO-OFDM systems in a frequency-selective fast fading channel, in which the channel is constant within each single OFDM block, and it may change independently from one OFDM block to another. We differentially encode the transmitted signals in the frequency domain within each

OFDM block so that differential decoding can be performed over subcarriers within each OFDM block. Differential space-frequency group codes that are investigated in this paper are known to have a simple maximum likelihood (ML) receiver [1], [4]. We have also embedded the interleaver design or subcarrier allocation method in the DSFC design criteria similar to [1]. The choice of subcarrier allocation vector is not chosen in an *ad hoc* fashion but justified by algebraic manipulations. This subcarrier allocation method is based on the assumption of a priori knowledge of delay profile of the channel at the transmitter, but not exact CSI. We also assume that fading channels is constant only within one OFDM block, and may change independently from one block to another.

The remainder of this paper is organized as follows. In Section II, the MIMO-OFDM system and channel model are presented. In Section III, we describe the DSFC design criteria, transmit-receive model, diversity order-coding gain, subcarrier grouping-starting signal and discuss about computational complexity. In Section IV, simulations are given, while in Section V, conclusions are presented.

*Notation:*  $\mathbb{E}$  denotes the expectation.  $\text{Tr}(\mathbf{X})$  denotes the trace of the matrix  $\mathbf{X}$ .  $|\mathbf{X}|$  is the determinant of the matrix  $\mathbf{X}$ .  $\|\mathbf{X}\|_F^2$  denotes the Frobenius norm of  $\mathbf{X}$  which is  $\|\mathbf{X}\|_F^2 = \text{Tr}(\mathbf{X}\mathbf{X}^\dagger) = \text{Tr}(\mathbf{X}^\dagger\mathbf{X})$ .  $\mathbf{I}_x$  is the identity matrix of order  $x$ .

## II. MIMO OFDM SYSTEM AND CHANNEL MODEL FORMULATION

In this paper, we consider a MIMO wireless communication system with  $T$  transmit antennas and  $R$  receive antennas. We assume that the antennas are spaced sufficiently far such that the channel between transmitter-receiver pair is quasi-static, wide-sense stationary uncorrelated scattering channel, whose impulse response is

$$h_{t,r}(\tau) = \sum_{l=1}^L h_{l,t,r} \delta(\tau - \tau_{l,t,r}), \quad (1)$$

where  $L = \lfloor B\tau \rfloor + 1$  is the total number of taps with  $B$  and  $\tau$  denoting the signal bandwidth and delay spread,  $h_{l,t,r}$  and  $\tau_{l,t,r}$  denote the  $l$ th path complex gain and time delay from transmitter  $t$  to receiver  $r$ , respectively. At the output of the OFDM demodulator, discrete-time frequency response of the channel is expressed in matrix form as

$$\mathbf{H}(n) = \sum_{l=1}^L \mathbf{H}_l e^{-j\frac{2\pi}{N}n\beta_l}, \quad n = 0, \dots, N-1, \quad (2)$$

where  $\mathbf{H}(n)$  and  $\mathbf{H}_l$  are both  $R \times T$ ,  $\mathbf{H}_l$  is the channel impulse response for the  $l$ th path,  $\beta_l = \tau_l/T_p$ ,  $T_u$  is the inverse carrier spacing or FFT interval,  $\Delta f = 1/T_u$  is the subcarrier spacing,  $T_p = T_u/N$  is the sampling period, and  $N$  denotes the number of subcarriers of the OFDM block. Note that the duration of one OFDM block is  $T_s = T_u + \Upsilon$ , where  $\Upsilon$  is the duration of guard interval.

We assume that the elements of  $\mathbf{H}_l$  for  $l = 0, 1, \dots, L-1$  are circularly symmetric zero-mean uncorrelated complex Gaussian random variables with variance  $\sigma_l^2$  and the channel is modeled as Rayleigh fading channel.

## III. DIFFERENTIAL SF GROUP CODES FOR MIMO-OFDM

### A. Design Criteria

In the following, we consider the situation when both the transmitter and the receiver has no knowledge of the CSI. Letting  $\mathbf{C}_G \triangleq \mathbf{G}\mathbf{D}$ , the pairwise error probability  $P(\mathbf{C}_G \rightarrow \mathbf{C}_{G'})$  for the differential transmission when the receiver erroneously decodes  $\mathbf{C}_{G'}$  when  $\mathbf{C}_G$  is actually transmitted is bounded by [4],[5]

$$P(\mathbf{C}_G \rightarrow \mathbf{C}_{G'}) \leq \prod_{r=1}^R \prod_{i=1}^{\text{rank}(\mathcal{V})} \frac{1}{1 + \frac{T\rho^2}{4(1+2T\rho)} \lambda_i(\mathcal{V})}, \quad (3)$$

where

$$\mathcal{V} = \mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G - \mathbf{C}_{G'}) \cdot \mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G - \mathbf{C}_{G'})^\dagger, \quad (4)$$

$\text{rank}(\mathcal{V})$  is the rank of  $\mathcal{V}$ ,  $\lambda_i(\mathcal{V})$  is the  $i$ th eigenvalue of the matrix  $\mathcal{V}$  and  $\rho$  is the SNR value. The  $Q \times TL$  matrix  $\mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G)$  has the following form: [1]

$$\mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G) = \begin{bmatrix} \alpha_1 \mathbf{A}_s^{\beta_1} \mathbf{C}_G & \alpha_2 \mathbf{A}_s^{\beta_2} \mathbf{C}_G & \cdots & \alpha_L \mathbf{A}_s^{\beta_L} \mathbf{C}_G \end{bmatrix} \quad (5)$$

where  $\mathbf{A}_s$  is the  $Q \times Q$  *delay rotation matrix* which is a function of the subcarrier allocation vector  $s$ :

$$\mathbf{A}_s = \begin{bmatrix} e^{-j\frac{2\pi}{N}s_0} & & & \\ & e^{-j\frac{2\pi}{N}s_1} & & \\ & & \ddots & \\ & & & e^{-j\frac{2\pi}{N}s_{Q-1}} \end{bmatrix}. \quad (6)$$

It can be observed that the codeword matrix in the Chernoff upper bound in ST codes [5] is replaced by the matrix  $\mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G)$ , which is termed as the *Krylov codeword* associated with the codeword  $\mathbf{C}_G$  [1]. Hence, the Chernoff upper bound on the PEP in Equation (3) is a function of the channel power profile  $\alpha$ , channel delay profile vector  $\beta$  and subcarrier allocation vector  $s$ .

For large  $\rho$ , the bound in Equation (3) is similar to the coherent bound shown in [1] except for a 3-dB loss in  $\rho$ . Therefore, it can be concluded that the differential transmission performs within 3 dB of coherent transmission, and the code design criterion is the same as coherent transmission similar to differential space-time modulation in [4],[5].

### B. Differential SF Code Transmit and Receive Model

Let  $\mathcal{C}$  be a space-frequency block code of size  $M$ :  $\mathcal{C} = \{\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_M\}$ , where each  $\mathbf{C}_m$  is a  $Q \times T$  matrix to be transmitted from  $Q$  subcarriers and  $T$  transmit antennas with  $Q \leq N$ . The  $(q, t)$ th element of  $\mathbf{c}_q$  is transmitted from subcarrier  $s_q$  and antenna  $t$  [11].

For a  $\mathbf{C}_k \in \mathcal{C}$  let  $\mathbf{C}_k = [\mathbf{c}_0, \mathbf{c}_1, \dots, \mathbf{c}_{Q-1}]^T$ , where  $\mathbf{c}_q$  is the length  $T$  vector to be transmitted from  $T$  antennas on subcarrier  $s_q$ . In the following  $N$  denotes the number of OFDM subcarriers. The OFDM modulator applies an  $N$ -point IFFT to  $N$  consecutive data symbols and then prepends a cyclic prefix (CP) of length  $L_{CP} \geq L$  (which is a copy of the

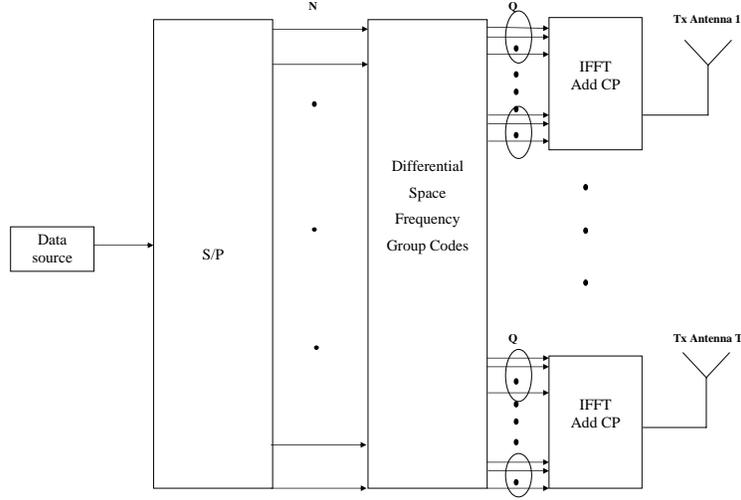


Fig. 1. MIMO-OFDM Transmitter for Differential Space Frequency Group Codes for  $T$  transmit antennas

last  $L_{CP}$  samples of the OFDM symbol) to the parallel-to-serial converted OFDM symbol. The receiver first discards the cyclic prefix, and then applies an  $N$ -point FFT to each of the  $R$  received signals. After FFT demodulation at the receiver, the signal at the  $s_q$ th subcarrier is subject to flat-fading and additive complex white Gaussian noise  $\mathbf{w}_q$ , which is statistically independent among different receiver antennas and different subcarriers, i.e.,  $\mathcal{E}(\mathbf{w}_q \mathbf{w}_q^\dagger) = \sigma_n^2 \mathbf{I}_R \delta[q - q']$ . The  $R \times 1$  signal vector  $\mathbf{y}_q$  received at  $R$  receiver antennas at the subcarrier  $s_q$  can be expressed as

$$\mathbf{y}_q = \sqrt{\rho/T} \mathbf{H}(s_q) \mathbf{c}_q + \mathbf{w}_q \quad q = 0, 1, \dots, Q-1, \quad (7)$$

where  $\rho$  is the average SNR at each receive antenna. Equation (7) can be rewritten as

$$\mathbf{Y}_k = \sqrt{\rho/T} \tilde{\mathbf{C}}_k \mathbf{H}_k + \mathbf{W}_k, \quad k = 1, \dots, N/Q \quad (8)$$

where

$$\mathbf{Y}_k = \begin{bmatrix} \mathbf{y}_0^T \\ \mathbf{y}_1^T \\ \vdots \\ \mathbf{y}_{Q-1}^T \end{bmatrix}, \tilde{\mathbf{C}}_k = \begin{bmatrix} \mathbf{c}_0^T & & & \\ & \mathbf{c}_1^T & & \\ & & \ddots & \\ & & & \mathbf{c}_{Q-1}^T \end{bmatrix} \quad (9)$$

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}(s_0)^T \\ \mathbf{H}(s_1)^T \\ \vdots \\ \mathbf{H}(s_{Q-1})^T \end{bmatrix}, \mathbf{W}_k = \begin{bmatrix} \mathbf{w}_0^T \\ \mathbf{w}_1^T \\ \vdots \\ \mathbf{w}_{Q-1}^T \end{bmatrix}.$$

Space-frequency group codes are multiantenna block codes that have a group structure like space-time group codes [1],[5]. Each  $Q \times T$  code matrix takes the form,

$$\mathbf{C}_k = \mathbf{G}_k \mathbf{D} \quad (10)$$

where  $\mathbf{G}_k$  belongs to a set of  $Q \times Q$  unitary generating

matrices  $\mathcal{G} = \{\mathbf{I}, \mathbf{G}_0, \mathbf{G}_0^2, \dots, \mathbf{G}_0^{M-1}\}$ , where

$$\mathbf{G}_0 = \begin{bmatrix} e^{-j \frac{2\pi}{M} \vartheta_0} & 0 & \dots & 0 \\ 0 & e^{-j \frac{2\pi}{M} \vartheta_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-j \frac{2\pi}{M} \vartheta_{Q-1}} \end{bmatrix} \quad (11)$$

and  $\mathbf{G}_k \mathbf{G}_k^\dagger = \mathbf{G}_k^\dagger \mathbf{G}_k = \mathbf{I}_Q$ ,  $\mathbf{D}$  is a fixed  $Q \times T$  complex matrix called the *initial or starting matrix* [1],[5]. Therefore, the set of all code matrices are defined in  $\mathcal{GD}$ .

Similarly to differential space-time modulation that is proposed in [5], we can also construct differential space-frequency modulation such that a sequence of messages  $\mathbf{G}_k \in \mathcal{G}$  is differentially encoded in a way similar to differential PSK across the OFDM subcarriers in a group manner as shown in Figure 1 for  $T$  transmit antennas. We assume that at  $k=0$ , we transmit  $\mathbf{C}_0 = \mathbf{D}$  to initialize the transmission. Thereafter, we encode the transmitted signals differentially in the frequency dimension within each OFDM block as follows:

$$\mathbf{C}_k = \mathbf{G}_k \mathbf{C}_{k-1}, \quad k = 1, \dots, N/Q \quad (12)$$

The group property guarantees that  $\mathbf{C}_k \in \mathcal{GD}$  whenever  $\mathbf{C}_{k-1} \in \mathcal{GD}$  [5].

We consider the differential decoding over two consecutive received matrices  $\mathbf{Y}_k$  and  $\mathbf{Y}_{k-1}$  for  $k = 1, \dots, N/Q$  as follows [8]. Since

$$\mathbf{Y}_k = \sqrt{\rho/T} \tilde{\mathbf{C}}_k \mathbf{H}_k + \mathbf{W}_k, \quad (13)$$

$$= \sqrt{\rho/T} \mathbf{G}_k \tilde{\mathbf{C}}_{k-1} \mathbf{H}_k + \mathbf{W}_k,$$

$$\mathbf{Y}_{k-1} = \sqrt{\rho/T} \tilde{\mathbf{C}}_{k-1} \mathbf{H}_{k-1} + \mathbf{W}_{k-1}, \quad (14)$$

so, we have

$$\mathbf{Y}_k = \mathbf{G}_k \mathbf{Y}_{k-1} + \sqrt{\rho/T} \tilde{\mathbf{C}}_k \Delta_k + \mathbf{W}'_k, \quad (15)$$

where  $\Delta_k = \mathbf{H}_k - \mathbf{H}_{k-1}$  is the channel difference matrix

between  $\mathbf{H}_k$  and  $\mathbf{H}_{k-1}$ , and  $\mathbf{W}'_k = \mathbf{W}_k - \mathbf{G}_k \mathbf{W}_{k-1}$  is a noise matrix whose each entry is an independent complex Gaussian random variable with mean zero and variance  $\sqrt{2}\sigma_n^2$  since  $\mathbf{G}_k$  is unitary.

Simple receivers for Equation (15) were proposed in [3],[4] if we assume  $\mathbf{H}_k \approx \mathbf{H}_{k-1}$  or the Frobenius norm of the channel difference  $\|\Delta_k\|_F$  is small enough such that  $\|\sqrt{\rho/T}\tilde{\mathbf{C}}_k\Delta_k\|_F$  is much less than  $\|\mathbf{W}'_k\|_F$ . Using this signal representation, the ML receiver for  $\mathbf{G}_k$  based on  $\mathbf{Y}_k$  and  $\mathbf{Y}_{k-1}$  is [4], [5],[8]:

$$\begin{aligned} \hat{\mathbf{G}} &= \arg \max_{\mathbf{G} \in \mathcal{G}} p(\mathbf{Y}_k | \tilde{\mathbf{C}}_k) \\ &= \arg \min_{\mathbf{G} \in \mathcal{G}} \|\mathbf{Y}_k - \mathbf{G}\mathbf{Y}_{k-1}\|_F \\ &= \arg \min_{\mathbf{G} \in \mathcal{G}} \text{Tr}\{(\mathbf{Y}_k - \mathbf{G}\mathbf{Y}_{k-1})(\mathbf{Y}_k - \mathbf{G}\mathbf{Y}_{k-1})^\dagger\} \\ &= \arg \max_{\mathbf{G} \in \mathcal{G}} \text{ReTr}\{\mathbf{G}\mathbf{Y}_k^\dagger \mathbf{Y}_{k-1}\} \end{aligned} \quad (16)$$

where "ReTr" denotes the real part of the trace

$$\text{ReTr}[A] = \frac{1}{2}(\text{Tr}[A] + \text{Tr}[A^\dagger]).$$

Notice that this receiver is of the same form with the correlation receiver [1, Eq. (10)], except with the unknown  $\mathbf{H}\mathbf{C}_{k-1}$  is estimated by  $\mathbf{Y}_{k-1}$ . When the differential encoder in Equation (12) is combined with the receiver in Equation (16), we say that  $\mathcal{G}\mathbf{D}$  is used for *differential transmission* (or *differential space-frequency modulation*) similar to [5] for differential space-time modulation.

### C. Diversity order and Coding Gain

We know that noncoherent and differential SFCs can potentially achieve the same maximum diversity order as coherent SFCs (i.e. the product of the number of transmit and receive antennas and the multipath channel channel length) over frequency-selective channels by appropriate code construction methods [2],[16],[18].

We can achieve the maximum transmit diversity  $TL$  by making the Krylov codeword  $\mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G - \mathbf{C}_{G'})$  full-rank (rank  $TL$ ) for any codeword pair  $\mathbf{C}_G$  and  $\mathbf{C}_{G'}$  [11]. We want to keep  $Q$  as large as possible in order to obtain larger diversity gain. On the other hand,  $Q$  decreases if we increase the number of SF codewords transmitted in parallel,  $N/Q$ . Therefore, we have to obtain a trade-off between rate and diversity gain [11]. If the number of channel taps is large, full diversity achieving differential SF group codes with  $Q \geq TL$  fails as shown in Section IV. Thus, we need to realize a trade off between the diversity order and the code rate especially in channels with high rms delay spread. For this reason less than full-diversity achieving differential SF group codes with  $Q \leq TL$  are used in this paper. The code rate is defined as,

$$\mathcal{R} = \left(\frac{1}{Q} - \frac{1}{N}\right) \log_2 M \text{ bits/subcarrier use.} \quad (17)$$

Notice that when  $Q \ll N$ , which is true in general, DSFC and coherent space frequency codes have approximately same rate.

In order to obtain the coding gain as maximum as possible, the minimum product of the nonzero eigenvalues of  $\mathcal{V}$ , which

is called the minimum *Krylov product distance*  $\Lambda_{\mathcal{K}}$ , has to be maximized. A SF code is said to be optimal if it is Krylov-full-rank for all pair of codes and has the largest Krylov product distance [1]. Krylov product distance is related to not only the SF code itself, but also the channel environments [11]. In general, space-frequency codes design aims at maximizing the diversity and coding gains. In this paper, coding gain is our primary goal because as shown in Section IV, diversity order does not monotonically improve the BER performance of differential space-frequency codes in channels with lots of taps. Therefore, we want to maximize the minimum Krylov product distance which is defined as [11]:

$$\begin{aligned} \Lambda_{\mathcal{K}} &= \min_{\mathbf{C}_G, \mathbf{C}_{G'} \in \mathcal{C}} \Lambda_{\mathcal{K}}(\mathbf{C}_G, \mathbf{C}_{G'}), \text{ where} \\ \Lambda_{\mathcal{K}}(\mathbf{C}_G, \mathbf{C}_{G'}) &= |\mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G - \mathbf{C}_{G'})^\dagger \cdot \mathbf{K}_{\alpha,\beta,s}^L(\mathbf{C}_G - \mathbf{C}_{G'})|^{1/TL} \end{aligned} \quad (18)$$

Similarly to [1], we can decouple the channel power profile from the rest of the variables in the Krylov Product distance to get the *effective Krylov product distance*  $\Lambda_{\mathcal{K}}^*$ , which is the product distance of  $\mathbf{K}_{\beta,s}^L(\mathbf{D})$ . The *effective Krylov product distance* is defined as [1]:

$$\Lambda_{\mathcal{K}}^* = \min_{\mathbf{G} \in \mathcal{G}, \mathbf{G} \neq \mathbf{I}} \left| \mathbf{K}_{\beta,s}^L(\mathbf{D})^\dagger (\mathbf{I} - \mathbf{G})^\dagger (\mathbf{I} - \mathbf{G}) \mathbf{K}_{\beta,s}^L(\mathbf{D}) \right|^{1/TL} \quad (19)$$

where

$$\mathbf{K}_{\beta,s}^L(\mathbf{D}) = \begin{bmatrix} \mathbf{A}_s^{\beta_1} \mathbf{d} & \mathbf{A}_s^{\beta_1+Z_1} \mathbf{d} & \dots & \mathbf{A}_s^{\beta_1+Z_{T-1}} \mathbf{d} \\ \dots & \mathbf{A}_s^{\beta_L} \mathbf{d} & \mathbf{A}_s^{\beta_L+Z_1} \mathbf{d} & \dots & \mathbf{A}_s^{\beta_L+Z_{T-1}} \mathbf{d} \end{bmatrix} \quad (20)$$

and  $Z = (Z_1, \dots, Z_{T-1})$  is the transmit delay vector which is chosen according to the delay profile  $\beta$  of the channel. The condition that must be satisfied between the delay profile  $\beta$ , the subcarrier group size  $Q$  and the transmit delay vector  $Z$  is found to be in [1] as,

$$\beta_l + Z_t - \beta_{l'} - Z_{t'} \neq kQ \quad \text{for } l \neq l' \text{ or } t \neq t' \text{ or both.} \quad (21)$$

### D. Starting signal $\mathbf{D}$ and subcarrier allocation vector $\mathbf{s}$

For  $T$  transmit antennas, the starting matrix  $\mathbf{D}$  can be chosen similarly in [1],

$$\mathbf{D} = \begin{bmatrix} \mathbf{d} & \mathbf{A}_s^{Z_1} \mathbf{d} & \mathbf{A}_s^{Z_2} \mathbf{d} & \dots & \mathbf{A}_s^{Z_{T-1}} \mathbf{d} \end{bmatrix} \quad (22)$$

where  $\mathbf{d}$  is the  $[Q \times 1]$  starting matrix

$$\mathbf{d} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad (23)$$

and  $\mathbf{A}_s$  is defined in Equation (6).

If the delay spread of the multiple paths is large with respect to the OFDM block period, i.e. the spectrum of the channel impulse response changes fastly, the ML detector proposed in [3],[4] fails because the channel frequency response over two adjacent subcarriers is not slowly changing. In this case, we are trying to create a smooth channel by distributing the

DSFCs over different subcarriers of MIMO-OFDM according to PDP of the channel so that the received signal matrix over consecutive groups can be decoded successfully.

The  $(q, t)$ th element of  $\mathbf{C}_m$  is transmitted from subcarrier  $s_q$  and antenna  $t$ , where the subcarrier group for each SF block  $\mathbf{s} = (s_0, s_1, \dots, s_{Q-1})$  is interleaved across the whole bandwidth, as [11],

$$s_q = s_0 + q \cdot N/Q \quad (24)$$

where  $s_0$  can take  $N/Q$  different values, one for each of the  $N/Q$  codewords transmitted in parallel:  $s_0 = 0, 1, \dots, N/Q - 1$ .

### E. Computational Complexity

We have employed subcarrier grouping as an essential part of the DSFC design. As compared to noncoherent space-frequency (SF) coding [2] where no subcarrier grouping is carried out, DSFC has a much lower complexity both in code design and in decoding. Our group code design does not depend on the number of subcarriers  $N$  whereas, the code search complexity of the noncoherent SF codes in [2] is exponential in  $N$ .

The complexity of the ML decoding is known to grow exponentially with number of antennas  $T$ , the data rate  $\mathcal{R}$  and the number of subcarriers per group [16]. In terms of design complexity, the code construction method in this paper reduces to choosing  $TL$  integers from a subset of  $0, 1, \dots, M - 1$  similar to [1]. Therefore, the complexity of DSFC is  $\mathcal{O}(\mathcal{M})$  since we need to obtain the decision metric for each of the  $M$  codewords in  $\mathcal{C}$ .

In our proposed construction, the decoding complexity grows exponentially with the codeword length since we need to calculate the correlator outputs for all possible codewords. On the other hand, the decoding complexity can be reduced by using short block length  $Q$  with subblock coding.

## IV. SIMULATION RESULTS

We simulated the proposed differential space-frequency group codes for a system with  $T=2$  transmit and  $R=1$  receive antennas. We consider a MIMO-OFDM system with transmission bandwidth  $B = 20$  MHz. The frequency bandwidth is divided into 64 subcarriers, yielding a subchannel spacing  $\Delta f$  of 312.5kHz. To make subchannels orthogonal, OFDM symbol duration is  $3.2\mu\text{s}$ . The guard interval duration is  $0.8\mu\text{s}$ , which is large enough to eliminate inter-symbol interference (ISI) and inter-carrier interference (ICI). We have non-line of sight (NLOS) Nokia rooftop wideband channel model in a suburban environment [11]. Its root mean-squared (rms) delay spread is about 49ns, and a 10-tap delay line model is built to model its average power-delay profile as shown in Table II [11].

TABLE I  
PARAMETERS ( $N=64, Q=4, T=2$ ) [1]

|      | $(M; \vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3)$ | $(Z_1; \Lambda_{\mathcal{K}}^*)$ |
|------|---|----------------------------------|
| BPSK | (2; 1, 1, 1, 1)   | (2; 5.97)                        |

Table I lists the best found parameters  $(\vartheta_0, \vartheta_1, \vartheta_2, \vartheta_3)$  and transmit delay vector  $Z_1$  that maximizes the *effective Krylov product distance*  $\Lambda_{\mathcal{K}}^*$ , defined in Equation (19) through exhaustive computer search [1].

In Figure 2, we compare the the differential space-frequency group codes using BPSK modulation with optimum coherent space-frequency group codes proposed in [1] when  $Q = 4$ , i.e. the diversity order is 4. From this figure, we can see that coherent detection outperforms the corresponding differential detection by 3 dB. On the other hand, implementing coherent detector requires robust channel estimation algorithms.

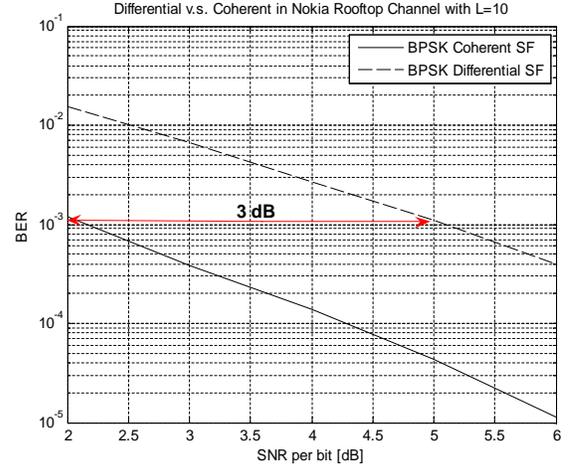


Fig. 2. Performance comparison of differential and coherent space-frequency codes in 10-ray nokia rooftop channel with r.m.s. delay spread of 49ns.

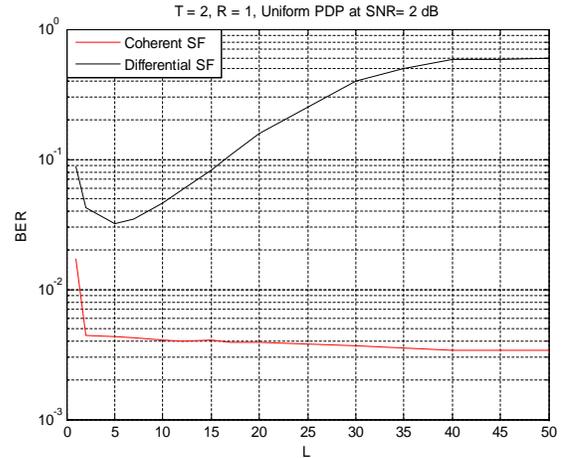


Fig. 3. Comparison of differential and coherent space-frequency codes in uniform PDP with  $\sigma_l^2 = 1/L$  ( $l = 0, 1, \dots, L - 1$ ) as a function of the number of channel taps at  $\rho = 2\text{dB}$

The effect of the additional channel taps in uniform power delay profile (PDP) is shown in Figure 3. Although a uniform PDP is not encountered in practice, the idea of using this model in the analysis is that, it is straightforward to choose the transmit delay vector that maximizes the coding gain when the channel has uniform PDP. The best found transmit delay vector parameters are  $Z = (L, L+1, \dots, L+T-2)$  when the channel

TABLE II  
NOKIA ROOFTOP CHANNEL 10-RAY POWER DELAY PROFILE

| Delay Profile ( $\mu\text{s}$ ) | 0      | 0.03   | 0.07   | 0.11   | 0.15   | 0.19   | 0.23   | 0.27   | 0.31   | 0.35   |
|---------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Power Profile                   | 0.7094 | 0.1782 | 0.0577 | 0.0310 | 0.0126 | 0.0060 | 0.0028 | 0.0011 | 0.0009 | 0.0004 |

delay parameter is uniform (i.e.  $\beta = (0, 1 \dots, L - 1)$ ) [1]. From Figure 3, we can observe that additional channel taps increases the diversity order, but simultaneously, the channel uncertainty increases due to an increasing number of unknown channel taps. Therefore, there is an *optimum diversity order* which is found to be similar to noncoherent space-frequency orthogonal codebooks in [2] and also to the capacity analysis in [7] and [19] for Single Input Single Output (SISO) wide-band channels. This effect is called the "oversampling" [7] in which the number of unknown channel coefficients becomes so high that the DSFCs can no longer cope with the channel uncertainty. We can define the *effective number of channel taps*,  $\hat{L}_{crit}$ , the optimum number of channel taps, (from a capacity point of view that maximizes the lower bound on capacity) similarly to [7] as

$$\hat{L}_{crit} = \frac{PT_c}{T}, \quad (25)$$

where  $T_c$  is the *coherence time* and  $P = B\rho$  is the signal power per receive antenna. If the channel has lots of taps, there is no need to extract full space-frequency diversity for noncoherent or differential space-frequency codes, because exploiting maximum achievable diversity order for space-frequency codes as in [2] fails if the receiver lacks CSI. This result is completely different from the coherent case of space frequency group codes [1] in which additional channel taps increase the BER performance due to the exploitation of frequency diversity present in MIMO-OFDM frequency-selective channels.

## V. CONCLUSIONS

In this paper, we concentrated on differential coding scheme across antennas and OFDM tones (space-frequency coding) within a single OFDM symbol. Differential space-frequency group codes are obtained from optimal coherent space-frequency group codes by constructing a simple differential SF code transmit and receive model which is called *differential transmission* of SF group codes. The performance of our differential space-frequency modulated codes is investigated in frequency-selective channels with multiple taps and it was shown that exploiting full space-frequency diversity in channels with multiple taps fails, which is found to be in contrast with the coherent SF codes. Compared to existing exponential complexity with data rate noncoherent SFCs, DSFC decoding allows higher data throughput and low decoding complexity without any significant diversity gain loss. The complexity of DSFC is found to be  $\mathcal{O}(\mathcal{M})$  since we need to obtain the decision metric for each of the  $M$  codewords in  $\mathcal{C}$ .

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